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THE UNIVERSITY OF ALBERTA

Long-Term Optimal Operation of Hydro-Power Systems

by

©

Ahmed Mohamed Atallah

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
AND RESEARCH IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF ELECTRICAL ENGINEERING

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FALL 1988

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THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **Long-Term Optimal Operation of Hydro-Power Systems**, submitted by Ahmed Mohamed Atallah, in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering.

Supervisor

External Examiner

Date:- August 16-1988

ABSTRACT

In this thesis a presentation has been made to illustrate the solution for the half monthly operating policy of a multireservoir tree-connected hydro-power system with what is believed to be one of the largest hydro electric nonlinear optimization problems attempted considering the number of variables and constraints.

The problem is formulated as a minimum norm problem and solved using functional analysis and minimum norm formulation techniques. The time period used is half a month; therefore, short range hydraulic and electro-technique effects are not taken into consideration.

The tree system is a general case of the reservoir topology which adequately specifies any system with any arbitrary topological arrangement. It is an improvement over the methods which deal with independent rivers (parallel connections) that have several reservoirs in series.

The problem is solved considering the generation from each reservoir as a linear function first and then a quadratic function of the storage times the discharge through the turbine. Also, tailwater elevation is first considered constant and then improved to be a nonlinear function of the total release. Moreover, the storage is initially considered to be a linear function of the forebay elevation, then improved to the actual case, where it is a nonlinear function of the forebay elevation and then tackled using a cubic spline curve fitting technique. Furthermore, all types of equality and inequality constraints are considered. First, only linear type constraints are considered. Then both linear and nonlinear types of constraints which meet all the requirements are used.

In this thesis we first maximized the total generation. Then we maximized the total generation shaped uniformly to the load while meeting hard constraints and balancing the violations of soft constraints if there are any violations.

The attractive feature of this technique is its ability to automatically produce the optimal solution while satisfying the system constraints. The technique combines both methodology and experience and overcomes the influence of the starting point which is a common problem for other techniques.

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List of Symbols and Abbreviations

ALCAN	Aluminum of Canada
AOP	Assured Operating Plan
BPA	Bonneville Power Administration
CFS	Cubic feet per second
CP	Critical period
CPU	Central processing unit time
CRC	Critical rule curves
$\bar{D}_k$	Maximum draft requirement for plant k
ECC	Energy Content Curves
$E_{k,i}$	Forebay elevation of plant k at the end of a period i
FELCC	Firm Energy Load Carrying Capability
FGFC	Full Gate Flow Constraints
$G_{k,i}$	Total genertion, MWh, for plant k durding a period i
HSSP	Hydro System Scheduling Problem
INF(I)	Vector of plants expected natural inflow during a period I.
$INF_{k,i}$	Expected natural inflow of plant k during a period i
$INFP_{j,i}$	The inflow potential energy of composite reservoir j in period i
$INFP_{k,i}$	The inflow potenital energy of reservoir k at period i
$INFP_{k,j,i}$	The inflow of water to reservoir k on river j during a period i
KSFD	10^3 foot-day per second (kilo second foot day)=86.4 E6 ft^3
L(I)	Matrix of plants constants durding a period I
m	Total number of periods

MCF	10^6 cubic feet
n	Total number of plants
P	Number of independent rivers in a system
PNCA	Pacific Northwest Coordination Agreement
q_k	Discharge rate from plant k
R(I)	Vector of plants constants during a period I
ROR	Run-of-River
S(I)	Vector of plants spill during a period I
$S_{j,i}$	The amount of potential energy spilt from composite reservoir j at period i
$S_{k,i}$	Spillage of plant k during a period i
$S_{k,j,i}$	Spillage of plant k on river j during a period i
s.t.	subject to
TVA	Tennessee Valley Authority
TWE	Tailwater Elevation
U(I)	Vector of plants discharge during a period i
$U_{j,i}$	The discharge potential energy of composite plants j at a period i
$\overline{U}_{j,i}$	Maximum potential energy of composite plants j at a period i
$\underline{U}_{j,i}$	Minimum potential energy of composite plants j at a period i
$U_{k,i}$	Discharge through the turbine for plant k during a period i
$\overline{U}_{k,i}$	Maximum discharge requirement of plant k during a period i

$\underline{U}_{k,i}$	Minimum discharge requirement of plant k during a period i
$\hat{U}_{k,i}$	Maximum discharge soft constraints of plant k during a period i
$U_{k,j,i}$	Discharge from plant k on river j during a period i
$\overline{U}_{k,j,i}$	Maximum discharge requirements of plant k on river j during a period i
$\underline{U}_{k,j,i}$	Minimum discharge requirement of plant k on river j during a period i
$UP_{k,i}$	Outflow potential energy of reservoirs k to n in period i
V(I)	Vector of plants constants during a period I
W(I)	Vector of plants variables during a period I
WB	Water Budget
WCF	Water Conversion Factor (watt hour per cubic feet)
x^*	Normed dual of x, x^* is a subset of X^*
X(I)	Vector of reservoirs contents during a period i
$\overline{X}_{j,i}$	Storage maximum potential energy of composite reservoirs j at period i
$\underline{X}_{j,i}$	Storage minimum potential energy of composite reservoir j at period i
$X_{k,i}$	Storage of reservoir k at the end of period i
$\overline{X}_{k,i}$	Maximum storage requirement of reservoir k at the end of period i
$\underline{X}_{k,i}$	Minimum storage requirement of reservoir k at the end of period i
$\hat{X}_{k,i}$	Maximum storage soft constraints of reservoir k at the end of period i

$X_{k,j,i}$	Storage of reservoir k on river j at the end of period i
$\overline{X}_{k,j,i}$	Maximum storage constraint of reservoir k on river j at the end of period i
$\underline{X}_{k,j,i}$	Minimum storage constraint of reservoir k on river j at the end of period i
$XP_{k,i}$	Total potential energy stored in reservoirs k to m at the end of period i
$\alpha_{k,i}$	Reservoir k constant [watt hour/ft^3] at period i
$\beta_{k,i}$	Reservoir k constant [watt hour/(ft^3)^2] at period i
$\gamma_{k,i}$	Reservoir k constant [watt hour/(ft^3)^3] at period i
ϕ_k	Reservoir k constant [feet]
ψ_k	Reservoir k constant [feet/(ft^3)]
ξ_k	Reservoir k constant [feet/(ft^3)^2]

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CHAPTER I

INTRODUCTION

Optimal operation of a hydro-electric power system means the allocation of the available hydraulic, pumped storage, thermal, nuclear, and other resources of the hydro-electric power system to the various time intervals of the period under consideration in such a way that the total system production cost is a minimum within limits permitted by the constraints. The constraints reflect physical limits, bank erosion considerations, coordination agreements among various ownerships, and multipurpose requirements such as irrigation, navigation, fishing, flood control, water quality recreational uses, and other purposes if any. Hydro-plants are considered as limited energy plants since their energy productions are subject to reservoir constraints and seasonal variations in rainfall. However, it is worthwhile to fully utilize all available hydro-energy in place of fossil energy because of its low production cost.

No two hydroelectric systems in the world are alike. The reasons for the differences are the natural differences in the watersheds, the differences in the man-made storage and release elements used to control the water flows, and the very many different types of natural and man-made constraints imposed on the operation of hydroelectric systems. River systems may be simple, with relatively few tributaries with dams in series along the river, or they may extend over vast multi-national areas, and include many tributaries and complex arrangements of storage reservoirs such as the Bonneville Power

Administration (B.P.A.) system. However, the one single aspect of hydroelectric plants that differentiates the coordination of their operation more than any other is the many and highly varied constraints.

Long term multireservoir scheduling is a complex problem. The problem is a constrained, nonlinear problem in which the objective function is nonlinear and the constraints are a mixture of linear and nonlinear relationships. This can be a difficult problem even for moderate dimensions; considering the number of variables and the practical limitations of computer storage and time, it is an extremely formidable one.

The problem is dynamic so that present decisions (reservoir releases) for one reservoir have an impact on future decisions for all reservoirs. Also, the optimal operating strategy for one reservoir depends not only on its own energy content, but also on the corresponding content of each one of the remaining reservoirs.

The problem is a highly stochastic problem in which major uncertainties are associated with the reservoir inflows, load, and unit availability.

Moreover, spatial and time correlation among hydro inflows is often high and must be modelled.

An additional complication arises from the nature of the inequality constraints. Some of these derive from properties of the physical systems and cannot be violated under any circumstances (hard constraints), whereas others are expressions of desired operating ranges which can be violated to some extent (soft constraints).

1.1 Preface

A hydroelectric generation system consists of rivers, tributaries, reservoirs (such as lakes or ponds), power houses and additional hydro facilities for power generation such as feeders and gates. Pipes, canals and rivers interconnect reservoirs and power houses. The natural inflows into the system are stored in the reservoirs.

Conventional hydro power units are classified as either controllable or run-of-river plants. Because of insufficient pondage, the run-of-river power house must take the water as it becomes available. On the other hand, a controllable hydro plant can exercise control on both the level of output and times of generation by manipulating the pondage, storing water at night and weekends and generating at maximum output at peak demand times.

A hydro unit usually has two associated levels: the forebay located in the upstream and the afterbay located in the downstream. The water flowing out from the power house is called the tailwater, and it is released to the afterbay through the tailrace, Fig. 1. The difference in the water surface elevation of the forebay and of the tailwater elevation is called the head of the power house, Fig. 2. If the tailrace is submerged in the afterbay, as is the case at a power house with a reaction turbine, the elevation of the tailwater is the surface elevation of the afterbay. On the other hand, if the tailrace is located upstream of the afterbay, as in some cases of power houses with an impulse turbine, the tailwater elevation remains constant.

At some power houses, changes in the surface elevations of the forebay and the afterbay may lead to significant head level variations.

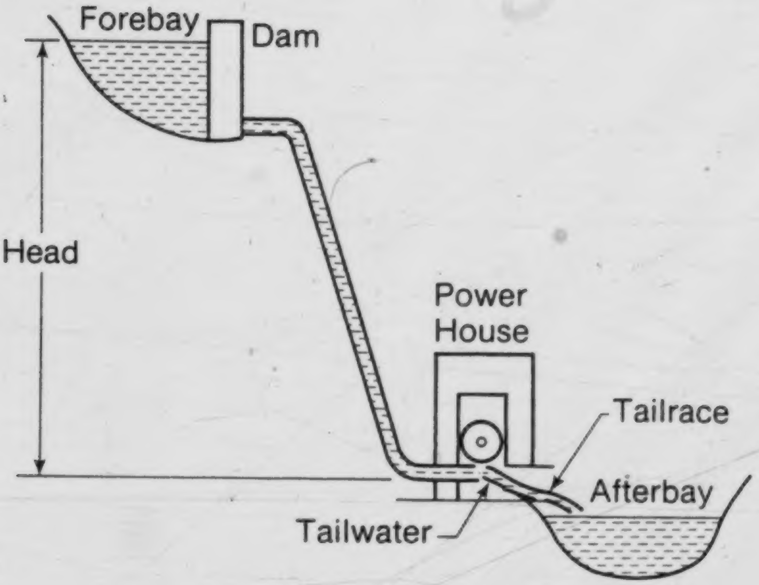

Forebay
Dam
Head
Power House
Tailrace
Afterbay
Tailwater

Figure 1 Schematic of a Power House with an Impulse Turbine.

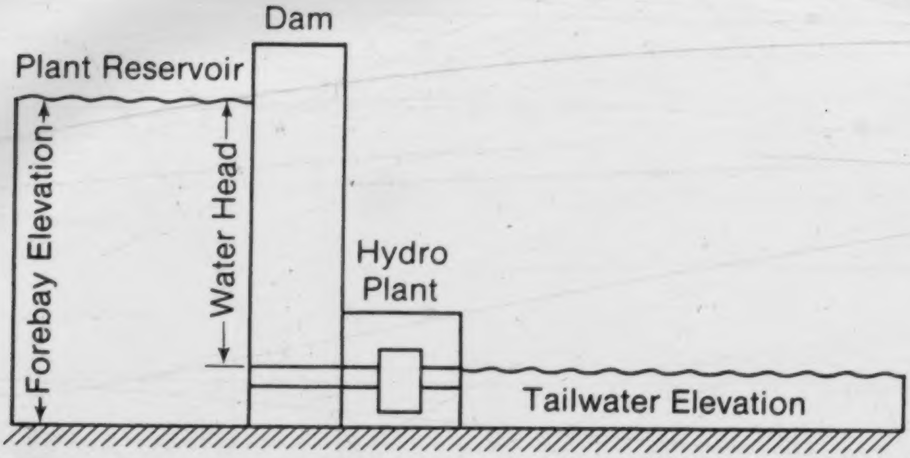

Dam
Plant Reservoir
Forebay Elevation
Water Head
Hydro Plant
Tailwater Elevation

Figure 2 Sketch of a Hydro Plant.

In such cases, the head is a function of the storage levels of the forebay and afterbay. Only in the cases where the forebay is located above the power house at such a high altitude that the surface level changes are small compared to the head, the changes in the forebay storage level may be neglected. At a run-of-river power plant, the forebay storage level is, in general, considered constant.

A particular hydro plant usually has n generating units. The generation of a unit, as a function of its flow, is zero up to a point q_1; then it increases up to the maximum generation q_3. Any flow more than q_3 is spillage which entails a decrease in generation due to an increase in tailwater elevation as in Fig. 3. For the plant the operating rule as the flow increases is to start a unit when the generation rate of the previously started unit becomes "too small", i.e., a little before the unit flow reaches q_3. In this study, each station will be reduced to a single equivalent input/output curve to reduce the number of variables in the optimization process.

The MW capacity of a hydro unit depends on the head and the flow through the power house. Typical curves showing the hydro generation capacity as a function of the water flow through the power house for different values of head for a reaction type unit are shown in Fig. 4. The maximum flow through a power house is also a function of the head; therefore, the maximum MW output of a unit is a function of the head only. This is indicated in Fig. 4 by the broken curve labeled maximum flow.

1.2 The Problem General Description

The primary objective in the operation of a hydroelectric

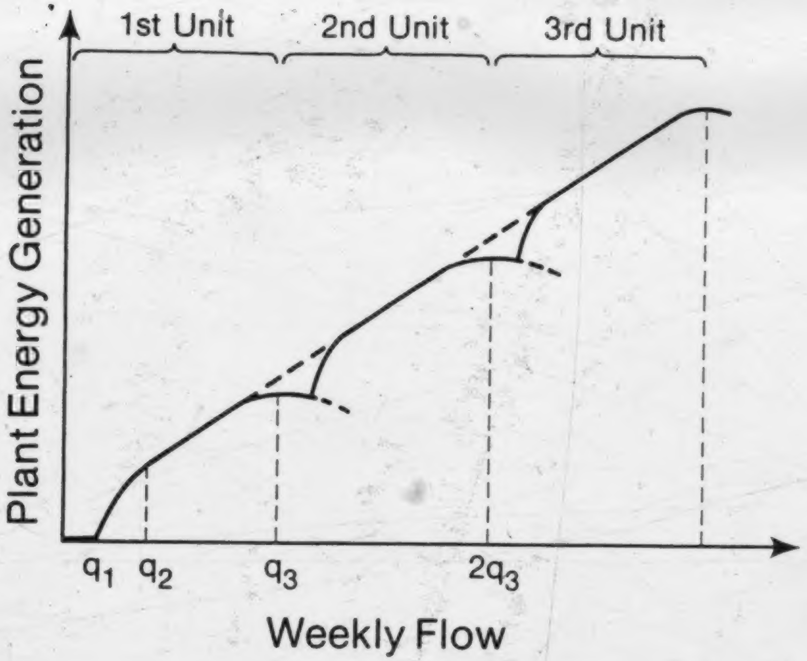


Figure 3 Plant Operation Rule for Energy Generation.

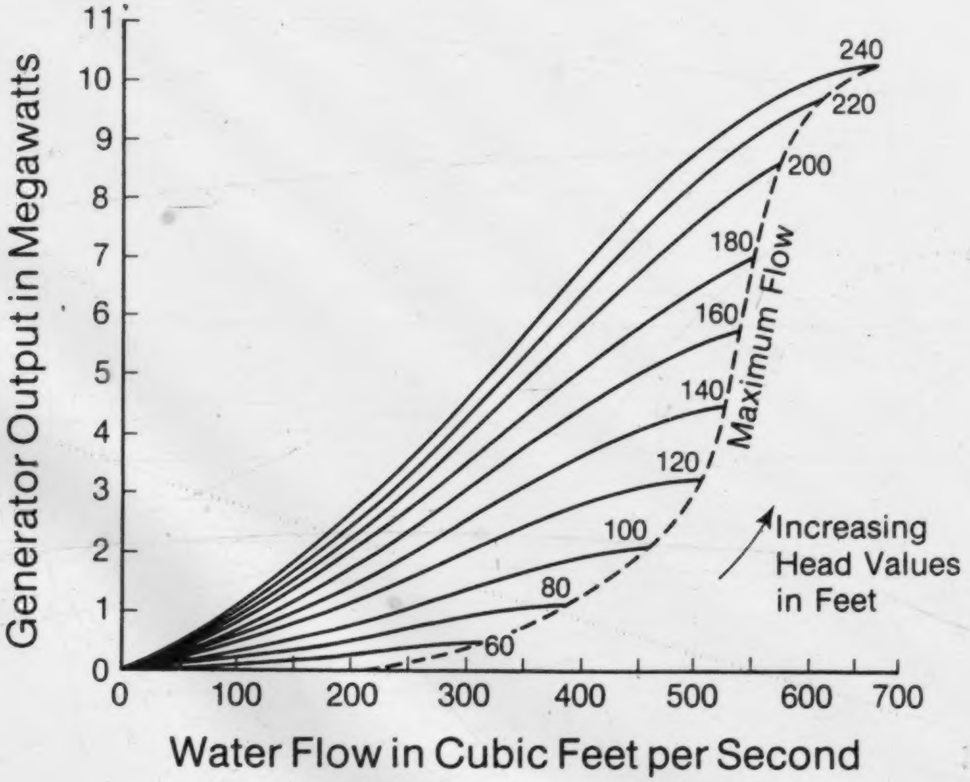


Figure 4 Hydro Generation Capacity as a Function of Water Flow Through the Power House for Different Values of Head.

generation system is to use the water in the most efficient way, while satisfying all the constraints imposed by legal and contractual obligations, physical characteristics and operating policies. The optimal water release schedules are determined at each reservoir so as to meet this goal. Due to the large-scale network structure of hydro systems of practical interest, determination of such schedules entails the solution of rather complex problems.

In this problem the initial amount of storage in each reservoir and the natural inflows into each stream during each period are assumed to be known. The forecasts of natural inflows are obtained using historical rainfall, river measurements and snow survey data. The demand for water, typically for irrigational purposes, is also assumed to be known at each location in each period. Evaporation and percolation losses are accounted for by deducting them from the forecasted sideinflows . The time period used in the modeling is half a month; therefore, short range hydraulic and electrotechnique effects are not considered. Also, due to the nature of the planning objective, deterministic critical period hydrological boundary type data are used.

The hydraulic system model is based primarily on field measured tables and on water balance equations. The latter are simple relationships between reservoir contents , total release and the inflow. The field measured tables basically consist of the following for each project:

i) forebay elevation as a function of the reservoir contents,

ii) tailwater elevation as a function of the total reservoir release,

iii) water-to-energy conversion factor as a nonlinear function of reservoir contents and turbine discharge

iv) maximum generation at each plant as a function of full-gate flow restriction which itself is a function of the head, Fig.4 .

1.3 Previous Work

The efficient utilization of hydro resources is of paramount importance in the planning and operation of a power system where the hydroelectric generation plants constitute a significant portion of the installed capacity and where a substantial reduction in the total system operating cost and in the risk of energy curtailments can be achieved by an appropriate management of the energy stored in various reservoirs. The hydro system scheduling problem then aims to determine the water releases from each reservoir and through each power house so as to optimize the total benefit of the hydro generated energy, while the various environmental, physical, legal and contractual constraints are satisfied.

Computing maximum energy capability of the hydro system is a stochastic, discrete-time problem principally concerned with seasonal management of reservoir storage. The problem is to determine, for all system reservoirs, a storage-management schedule which results in maximum energy capability for the system (total generation averaged over all periods in the whole critical period horizon) with system constraints satisfied and with acceptable uniformity in the surplus of generation over load during each period.

Solutions to this maximization problem have been determined in the past by skilled engineers using digital computer simulation programs and "cut and try" methods wherein reservoir storage-management schedules are successfully modified. This approach has become cumbersome with present day requirements to deal with such as 88 reservoirs and run-of-river plants and with adverse stream-flow periods of a 4 years long (96 periods).

Many other techniques for obtaining optimal operation have been applied and yet no completely satisfying solution has been obtained, since in every publication the problem has been over-simplified in order to cope with the dimensionality of the problem. Christensen, El-Hawary, and Soliman [1,2,70] and Rosenthal [60] have presented comprehensive surveys for the problem of multireservoirs, multiperiod deterministic or stochastic inflow, and the nonseparable benefit.

The two prominent approaches that have been used in solving the problem are nonlinear programming and dynamic programming.

Nonlinear programming techniques are developed for models with nonseparable benefits. Gagnon et.al. [9,50,61] and Hicks et.al. [7] worked with the Bonneville Power Authority, Hanscon et.al. [63] worked with Hydro-Quebec, Divi et.al. [64] worked with ALCAN (Aluminum of Canada) system and Rosenthal [49] with Tennessee Valley Authority (TVA). All but one of these applications required the model to handle a general network topology (any arbitrary topological configuration) for the reservoir system. The exception to this requirement was ALCAN, whose reservoirs exhibit only series arrangements.

The selected measure in TVA and Hydro-Quebec studies [49,63] was the savings of thermal fuels that results from hydroelectric generation. In the ALCAN study [64], the measure of benefit was the potential energy in the system at the end of the planning period. In the Bonneville study [7,9,50,61], the measure was a weighted sum of (i) the proportion of power load met with hydro, (ii) the uniformity of load deficits, and (iii) the violation in certain "soft" constraints.

The energy production as a function of volume of outflow is taken as a constant independent of the head [7,9,50,61] or as piece-wise polynomials, with two pieces; one piece represents energy production from outflows that are directed entirely to the turbines, the other piece is needed to account for spills i.e., the situations in which outflows exceed turbine capacities. The ALCAN research team [64] used a cubic spline to smooth out the breaks between the two pieces.

Hicks et.al. [7] transform the general nonlinear program formulation of the hydro system scheduling problem (HSSP) into an optimization problem with a nonlinear objective and only linear constraints. All the nonlinear constraints are expressed as penalty terms and are added to the original objective function. This approach is considered as the first successful attempt to solve a large HSSP with a nonlinear optimization technique [68]. However, the computational times reported are rather lengthy.

Hanscom et.al. [63] have employed a simpler model for a large scale nonlinear programming problem without any nonlinear constraints. They solved the large-scale scheduling problem in less time compared to that taken by Hicks et.al. [7].

Ikura and Gross [68] present a hydro scheduling problem formulation and a method of solution to this problem as a nonlinear programming problem. The maximum turbine discharge constraint is approximated by a piece-wise linear function. A nonlinear constraint for forced spill is treated as a penalty factor. The resulting problem has a nonlinear objective function with linear constraints and is solved using standard mathematical programming packages. A network flow algorithm is used to provide a good starting point for the standard programming packages. They [68] also have distinguished between forced and controllable spills which results in my opinion in an unnecessarily complex model.

Also Ikura and Gross [68] found that using a good starting point with a suitable linear network flow algorithm saves about 50% of the solution time.

The nonlinear programming algorithms [7,9,26,28, 47,49,50,54,61,63,68] that have been applied to the multireservoir model are fairly efficient in comparison with the dynamic programming approaches. Nevertheless, it does not appear likely that the nonlinear programming methods are adoptable for handling stochastic inflows, because the computational effort already expended in solving the deterministic problems is quite large using the nonlinear programming technique.

Dynamic programming approaches [14,15,16,17,18,21,51,52,62] to multireservoir models have as much difficulty as the nonlinear programming in accommodating stochastic inflows solution. Linear and nonlinear dynamic programming solution methods were used with relatively small systems, or after dividing the given system into small parts or after applying different methods of:

i) decomposition [15,16,25,30],

ii) aggregation [17,18,29],

iii) aggregation/decomposition [18], and

iv) successive approximations [52].

Murray and Yokowitz [62] convert the HSSP with a high dimensional state space into a sequence of problems with lower dimensional state spaces. Their technique helped in running the hydro scheduling problem with a memory and computational requirement that grows only as n^2 and n^3, respectively, instead of growing exponentially with n as in almost all the cases of dynamic programming. n is the number of reservoirs in the optimized project.

Turgeon [18] presents and compares two possible manipulation methods for solving the optimal operation of multireservoir power systems with stochastic inflows. The first, called the one-at-a-time method, consists in breaking up the original multivariable problem into a series of one-state variable subproblems that are solved by dynamic programming. The final result is an optimal local feedback operating policy for each reservoir. The second method, called the aggregation/decomposition method, consists in breaking up the original n-state variable stochastic optimization problem into n stochastic optimization subproblems of two-state variables that are also solved by dynamic programming. The final result is a suboptimal global feedback operating policy for the system of n reservoirs.

Other methods, Grygier and Stedinger [21] and Dillor et.al. [56], have simplified the problem by converting the highly nonlinear problem into a linear one and solving it using a kind of linear programming technique. Marino and Loaiciga [26], have used the quadratic programming technique to solve the problem of management of the Central Valley Project.

Christensen and Soliman [23,24,31,32,33,34,35,38,39,41], used functional analysis and the minimum norm formulation to solve the hydro system scheduling problem for relatively small systems (six reservoirs maximum). They considered constant tailwater elevation and only linear types of equality and inequality constraints. Also, they considered a linear relationship between reservoir content and forebay elevation. The water conversion factor is considered constant, linear function of the storage, linear function of forebay elevation, or quadratic function of the storage.

1.4 Scope of the Thesis

In this thesis, functional analysis and the minimum norm formulation technique have been used to maximize the total energy capability for the long-term problem of hydro power systems. The algorithm uses the general tree connection of reservoir and run-of-river plants which can fit any arbitrary topological configuration. Calculations have been made for the critical period (CP) which represents a historical stream record during which the natural inflows to the hydro system are the smallest on record. The optimization problem is described and formulated as the optimal control of a multivariable model in which the state and control variables are constrained by a set of equality and inequality constraints to satisfy the multipurpose stream use requirements. Lagrange and Kuhn-Tucker multipliers are used to adjoin these constraints to the objective function.

Chapter II presents a mathematical background. Chapter III is devoted to brief descriptions of the two system models used to apply the algorithm, the nine reservoir system and the B.P.A. hydro system. It summarizes some of the pitfalls that any optimization technique will encounter. In Chapter IV, the optimization algorithms are described and applied to the two models. In this chapter the water conversion factor (WCF) is considered as a linear function of the storage, the storage is considered as a linear function of the forebay elevation, the tailwater elevation is considered constant, and the maximum draft constraints are neglected. In Chapter V, the optimization technique is described and applied to the B.P.A. hydro system. In this chapter, the generation from each reservoir is considered as a quadratic function of the storage times the discharge through the turbine. The storage is considered as a nonlinear function of the forebay elevation and solved using the cubic spline curve fitting. Tailwater elevation is considered as a nonlinear function of the total release and calculated using the cubic spline curve fitting, and the maximum draft constraints are considered which result in a nonlinear state dependent type of inequality constraint. In Chapter VI the load-following scheduling problem is presented considering all the factors taken into account in the previous chapter and then applied to the B.P.A. system. Impressive results were obtained which show the strength of our technique.

CHAPTER II

AN APPROACH TO FUNCTIONAL ANALYSIS AND OPTIMIZATION TECHNIQUES

During the past twenty years mathematics and engineering have been increasingly directed towards problems of decision making in optimizational systems. This trend has been inspired primarily by the significant economic benefits which often result from a proper decision concerning the distribution of expensive resources, and by the repeated demonstration that such problems can be realistically formulated and mathematically analyzed to obtain good decisions.

The arrival of high-speed digital computers has also played a major role in the development of the science of decision making. Computers have inspired the development of larger systems and the coupling of previously separate systems, thereby resulting in decision and control problems of correspondingly increased complexity. At the same time, however, computers have revolutionized applied mathematics and solved many of the complex problems they generated.

Any problem investigated in an optimization analysis will have as its objective the improvement of the system or systems. It should be fairly obvious that, in order to improve any system, it is essential that at least one solution be obtainable for that system. In other words, by defining the inputs to a system, we can find the resulting output, if this is possible, if not then we cannot design or operate or control the system.

Usually, no single answer is normally found to any problem, and it is therefore necessary to choose the "best" solution for a given problem from the multitude of possible solutions. How can this be achieved? First, it is necessary to define the objective of the study. This may vary from one problem to another, but in our applications it is economic. Such economic aims may be a maximum power generation or minimum deficit. By the optimal we mean that such an objective has been maximized to the stage that no further improvement can be attained from any other study.

Mathematical optimization consists in determining values of variables to maximize or minimize an objective function. In many optimization problems the variables are also required to satisfy constraining equations or inequalities. Optimization problems often arise from the need to determine parameters of a model so that the model best fits measured data according to some criterion. Many optimization problems also arise from the desire to determine a system's operating policy which is in some sense best. In this setting, the equations which constitute the model of the system act as constraints on the optimization. Also, many of the physical limitations sometimes introduce additional constraints into optimization problems, often in the form of inequalities. The objective functions and the constraints functions may be arbitrary nonlinear functions.

This chapter consists of a collection of mathematical topics which serves as a basis for the next chapters.

2.1 The Fundamental Concepts and Methods of Functional Analysis

The fundamental concepts and method of functional analysis have gradually emanated from some of the oldest domains of analysis like calculus of variations, theory of differential equations, theory of approximation of functions, numerical analysis and in particular theory of integral equations [71]. This development of functional analysis has made rapid progress during the last decades.

Our objective in this section is to state an important minimum norm result which plays a crucial part in the solution of problems treated in this work. Before we do this, a brief discussion of relevant concepts from functional analysis is given.

2.1.1 **Linear Spaces [71]**

A set X is called a linear space if addition and scalar multiplication are defined on X and the following rules hold:

(1)

addition is commutative, i.e.,

$x+y=y+x$ for any x and y in X,

(2) addition is associative, i.e.,

$(x+y)+z=x+(y+z)$ for any x,y,z in X,

(3) there exists an element $0 \epsilon X$ (the zero element) such that $x+0=x$ for any $x \epsilon X$,

(4) for any $x \epsilon X$ there exists an element $y \epsilon X$ (the inverse of x) such that $x+y=0$,

(5) $1x=x$ for any $x \epsilon X$,

(6) $\alpha(\beta x)=(\alpha\beta)x$ for any $x \epsilon X$ and any scalar α and β,

(7) $(\alpha+\beta)x=\alpha x+\beta x$ for any $x \epsilon X$ and any scalar α and β,

(8) $\alpha(x+y)=\alpha x+\alpha y$ for any $x,y \epsilon X$ and any scalar α,

A linear space is called real or complex according to whether the scalars are the real or complex number system. The elements of a linear space are called vectors.

2.1.2 Normed Linear Spaces

A linear space is called a normed linear space if a rule exists which associates with every element, $x \in X$, a real number called the norm of an element x and denoted by $||x||$. This rule must obey the following conditions (norm axioms):

(1) $||x|| \geq 0$ and $||x|| = 0$ only if $X = 0$,

(2) $||x+y|| \leq ||x|| + ||y||$,

(3) $||\alpha x|| = |\alpha| . ||x||$, α is any scalar value.

The non-negative real number $||x||$ can be thought of as the length of the element x.

Since the same standard of comparison should be used to measure distances as well as lengths in a normed linear space, we can define the metric of the space by

$$\rho(x,y) = ||x-y||$$

2.1.3 Inner Products

The length $||x||$ of the vector x is defined by

$$||x|| \triangleq \sqrt{x.x} \quad \text{and}$$

$$x.y \triangleq ||y|| (||x|| \cos\phi)$$

where $||x|| \cos\phi$ is the length of the projection of x on y. Following are the properties of the dot product:

(1) length is non-negative, that is

$x.x\geq 0$ with equality if and only if $x=0$

(2) the magnitude of ϕ (or $\cos\phi$) is independent of the order of

x and y; that is $x.y=y.x$

(3) the length of cx equals $|c|$ times of length x for any scalar c; that is, $||cx||=|c|.||x||$

(4) distribution, i.e.,

$$(x_1+x_2).y = x_1.y+x_2.y$$

Definition: An inner product (or scalar product) on a real (or complex) vector space is a scalar-valued function $\langle x,y\rangle$ of the ordered pair of vectors x and y such that:

(1) $\langle x,x\rangle \geq 0$ with equality only if $x=0$

(2) $\langle x,y\rangle = \overline{\langle y,x\rangle}$ (the bar denotes complex conjugation)

(3) $\langle c_1x_1+c_2x_2,y\rangle = c_1\langle x_1,y\rangle+ c_2\langle x_2,y\rangle$

when $\langle x,y\rangle$ is real, we can define the angle ϕ between x and y by

$$\cos\phi = \frac{\langle x,y\rangle}{||x|| \quad ||y||}$$

Also, if $\langle x,y\rangle=0$, x and y are said to be orthogonal

If $\langle x,y\rangle = \pm ||x||\cdot||y||$, x and y are said to be collinear

Inner product space is called pre-Hilbert space.

2.1.4 Cauchy Sequence

An infinite sequence $[y_n]$ from an inner product space is called a Cauchy Sequence if $||y_n-y_m||\to 0$ as $n,m\to\infty$. Hence a Cauchy-

Sequence is a convergent sequence.

2.1.5 Complete Spaces

A metrix space X is said to be complete if every Cauchy (convergent) sequence from X converges to a limit in X (an element of X).

- A complete inner product space is called Hilbert space.
- Every finite-dimensional inner product space is complete.
- Banach spaces are complete metric linear spaces.
- A Hilbert space is a Banach space whose norm is induced by an inner product.

2.1.6 Hilbert Space

A Hilbert Space is a special form of normed space having an inner product defined. Hilbert spaces, equipped with their inner products, possess a wealth of structural properties generalizing many of our geometrical insights for two and three dimensions. The concepts of Fourier series and least-squares minimization all have natural settings in Hilbert space.

2.1.6.1 The Hilbert Space l_2

The space l_2 is the inner product of the vectors

$$x=(x_1,x_2,\ldots,n_n,\ldots) \text{ and } y=(y_1,y_2,\ldots,y_n,\ldots)$$

is defined by

$$\langle x,y\rangle = \sum_{n=1}^{\infty} x_n.\overline{y}_n$$

The usual norm and metric on l_2 are

$$||x||=(\langle x,x\rangle)^{1/2}=(\sum_{i=1}^{\infty}|x_i{}^2|)^{1/2}$$

$$\rho(x,y)=||x-y||=(\sum_{i=1}^{\infty}|x_i-y_i|^2)^{1/2}$$

2.1.6.2 The Hilbert Space $L_2(a,b)$

If $x(t)$ and $y(t)$ are functions from the space $L_2(a,b)$, the inner product,

$$\langle x,y\rangle=\int_a^b x(t)\overline{y}(t)dt$$

The norm and metric on $L_2(a,b)$ are given by

$$||x||=(\langle x,y\rangle)^{1/2}=[\int_a^b |x(t)|^2dt]^{1/2}$$

$$\rho(x,y)=||x-y||=[\int_a^b |x(t)-y(t)|^2dt]^{1/2}$$

2.1.6.3 The Hilbert Space $L_2(a,b;\mu)$

If $\mu(t)$ is any function $\mu(t)>0$ over the interval (a,b), then a valid inner product would be

$$\langle x,y\rangle=\int_a^b \mu(t)x(t)\overline{y}(t)dt;$$

in this case

$$||x||=[\int_a^b \mu(t)|x(t)|^2dt]^{1/2}$$

$$\rho(x,y)=||x-y||=[\int_a^b \mu(t)|x(t)-y(t)|^2dt]^{1/2}$$

2.1.6.4 The Hilbert Space of Random Variables [37,59]

Let $P(\xi)$ be the probability that the random variable x assumes a value less than or equal to the number ξ. The expected value and the variance of a discrete random variable x, denoted by μ_x and Var(x), is defined by

$$E[x]=\mu_x=\sum_x x(\xi)P(\xi)$$

and $\mathrm{Var}(x)=||x-E(x)||^2 = E(x^2)-E^2(x)$

where $\sum_x$ means the sum over all x values.

Given a finite collection of real random variables $[x_1,x_2,\ldots,x_n]$, we define their joint probability distribution P as

$$P(\xi_1,\xi_2,\ldots,\xi_n)=\mathrm{Prob.}(x_1\leq\xi_1,\ x_2\leq\xi_2,\ldots,x_n\leq\xi_n)$$

i.e. the probability of the simultaneous occurrence of $x_i\leq\xi_i$ for all i. The expected value of any discrete function g of x_i's is defined by

$$E[g(x_1,x_2,\ldots,x_n)]=\sum_x g(\xi_1,\xi_2,\ldots,\xi_n).P(\xi_1,\xi_2,\ldots,\xi_n)$$

Two random variables x_i, x_j are said to be uncorrelated if

$$E(x_i,x_j) = E(x_i).E(x_j)$$

To construct a Hilbert space of random variables let $[y_1,y_2,\ldots,y_m]$ be a finite collection of random variables with $E[y_i{}^2]<\infty$

for each i. A Hilbert space H that consists of all random variables is a space of linear combinations of the y_i's. The inner product of two elements x,y in H is defined as

$$\langle x,y\rangle = E(xy)$$

and if $x = \sum \alpha_i y_i$, $y = \sum \beta_i y_i$ then

$$E(x,y) = E[(\sum_i \alpha_i y_i).(\sum_i \beta_i y_i)]$$

The space H is a finite-dimensional Hilbert space with dimensions equal to at most m. If in the Hilbert space H each random variable has an expected value equal to zero, then two vectors x,z are orthogonal if they are uncorrelated:

$$\langle x,z\rangle = E(x).E(z) = 0$$

The concept of a random variable can be generalized in an important direction. An n-dimensional vector-valued random variable x is an ordered collection of n scalar-valued random variables. Notationally, x is written as a column vector

$$x = \text{col.}(x_1, x_2, \ldots, x_n)$$

x in the above equation is referred to as a random vector.

A Hilbert space of random vectors can be generated from a given set of random vectors in a manner analogous to that for random variables. Suppose $\{y_1,y_2,\dots,y_m\}$ is a collection of n-dimensional random vectors. Each element y_i has n components y_{ij}; $j=1,2,\dots,n$ each of which is a random variable with finite variance. We define the Hilbert space H of n-dimensional random vectors as consisting of all vectors whose components are linear combinations of the components of the y_i's. Thus an arbitrary element y in this space can be expressed as

$$y=K_1.y_1+K_2.y_2+\dots+K_n.y_n$$

where the K_i's are real nxn matrices.

If x and z are elements of H, we define their inner product as

$$<x,z> = E(\sum_{i=1}^{n} x_i z_i)$$

which is the expected value of the n-dimensional inner product. A convenient notation is

$$<\mathbf{x},\mathbf{z}> = E(\mathbf{x}^T\mathbf{z})$$

We refer to E(x) as the mean of the random variable x.

The variance of x is defined by

$$\mathrm{var}(x) \triangleq E\,||x-E(x)||^2=E(x^2)-E^2(x)$$

The covariance between x and y is defined by

$$\mathrm{cov}(x,y) \triangleq E\langle x-E(x),y-E(y)\rangle = E(x.y)-E(x)E(y)$$

The random variables x and y are said to be uncorrelated if $\mathrm{cov}(x,y)=0$ If x and y are uncorrelated, then

$$\mathrm{var}(x+y) = \mathrm{var}(x) + \mathrm{var}(y)$$

2.1.7 The Minimum Norm Theorems [72]

The first optimization problem is this: Given a vector x in a pre-Hilbert space X and a subspace M in X, find the vector $m \in M$ closest to x in the sense that it minimizes $||x-m||$. Of course, if x itself lies in M, the solution is trivial. In general, however, three important questions must be answered for a complete solution to the problem. First, is there a vector $m \in M$ which minimizes $||x-m||$, or is there no m that is at least as good as others? Second, is the solution unique, and third, what is the solution or how is it characterized?

Theorem 1

Let X be a pre-Hilbert space, $_m$ a subspace of M, and x an arbitrary vector in X. If there is a vector $m_o \in M$ such that

$$||x-m_o|| \leq ||x-m|| \quad \text{for all } m \in M,$$

then m_o is unique. A necessary and sufficient condition that $m_o \in M$ be a unique minimizing vector in M is that the error vector $x-m_o$ be orthogonal to M.

We still have not established the existence of the minimizing vector. We have shown that if it exists, it is unique and that $x-m_o$ is orthogonal to the subspace M. By slightly strengthening the hypotheses, we can also guarantee the existence of the minimizing vector.

Theorem 2

Let H be a Hilbert space and M a closed subspace of H. Then, corresponding to any vector $x \epsilon H$, there is a unique vector $m_o \epsilon M$ such that $||x-m_o|| \leq ||x-m||$ for all $m \epsilon M$. Furthermore, a necessary and sufficient condition that $m_o \epsilon M$ be the unique minimizing vector is that $x-m_o$ be orthogonal to M.

Now we consider the question of determining a vector in a subspace M of a normed space which best approximates a given vector x in the sense of minimum norm.

We have that if M is a closed subspace in Hilbert space, there is always a unique solution to the minimum norm problem and the solution satisfies an orthogonality condition. Furthermore, the projection theorem (theorem 2) leads to a linear equation for determining the unknown optimizing vector. The following two theorems contain more information than the projection theorem.

Theorem 3

Let x be an element in a real normed linear space X, and let d denote its distance from the subspace M. Then,

$$d = \inf_{m \epsilon M} ||x-m|| = \max_{\substack{||x^*|| \leq 1 \\ X^* \epsilon M^{\perp}}} \langle x, x^* \rangle$$

where the maximum on the right is achieved for some $x_o^* \epsilon M^{\perp}$. If the infimum on the left is achieved for some $m_o \epsilon M$, then x_o^* is aligned with $x-m_o$.

Theorem 4

Let x be an element of a real normed linear vector space X, and let M be a subspace of X. A vector $m_o \varepsilon M$ satisfies $||x-m_o|| \leq ||x-m||$ for all $m \varepsilon M$ if and only if there is a nonzero vector $x^* \varepsilon M^{\perp}$ aligned with $x-m_o$.

Theorem 5

Let M be a subspace in a real normed space X. Let $x^* \varepsilon X^*$ be a distance d from $M^{\perp}$. Then,

$$d = \min_{m^* \varepsilon M^{\perp}} ||x^* - m^*|| = \sup_{\substack{x \varepsilon m \\ ||x|| \leq 1}} \langle x, x^* \rangle$$

where the minimum on the left is achieved for $m_o^* \varepsilon M^{\perp}$. If the supremum on the right is achieved for some $x_o \varepsilon M$, then $x^* - m_o^*$ is aligned with x_o.

Theorem 5 guarantees the existence of a solution to the minimum norm problem if the problem is appropriately formulated in the dual of a normed space.

Notations

- $M^{\perp}$ is the orthogonal component to M
- inf means infimum or the greatest lower bound
- x^* is the normed dual of x
- A set P is said to be closed if $P = \overline{P}$

2.2 Lagrange Multipliers

These specify the magnitudes of the forces exerted by the constraints; that is, they indicate how hard each constraint is working. The importance of Lagrange multipliers is that they indicate the sensitivity of the minimum (maximum) value of the objective function to changes in each of the constraints.

CHAPTER III

THE SYSTEM UNDER STUDY

There are two systems which have been used for applying the mathematical technique. The first one is a moderate system composed of nine storage projects connected like a tree, the general case, Fig. 5. The other system is the B.P.A. system Fig. 6, a huge system composed of 51 run-of-river (ROR) type projects and 37 storage type projects on the Columbia river from Mica Dam to the Pacific Ocean. Fig. 7, presents a schematic diagram of the system.

In this chapter a description of the critical period hydro optimization problem of the B.P.A. system and of some of the obstacles (that any optimizer of that enormous system encounters) will be presented. Then a short description of each project in the B.P.A. system and the relevant necessary table of constraints will be touched upon lightly. Finally the nine storage projects system will be presented briefly.

3.1 Critical Period

The Critical Period (CP) refers to the period during which at-site stream flow decreased to its lowest rate in history. The stream flows during the critical period are called "Critical period stream flows". The total generation during the CP defines the maximum firm power generation from that system. The optimal generation during that period is that which maximizes the objective function and of course does not violate any of the given hard constraints of the whole system.

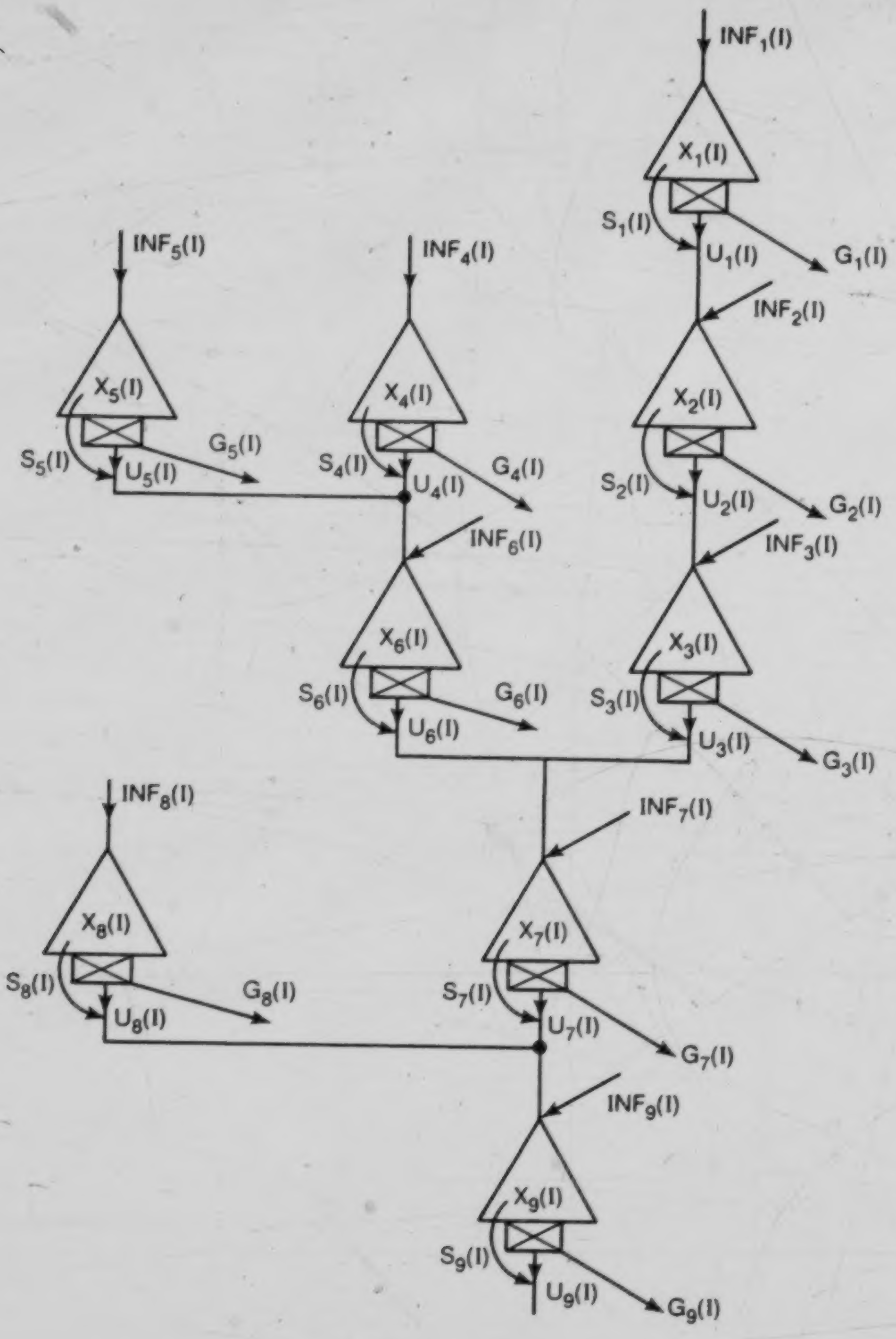
CP regulations are required by the Pacific Northwest Coordination

Agreement (PNCA) to determine the Firm Energy Load Carrying Capability (FELCC) of the system and to define energy content curves (ECC) and critical rule curves (CRC) for guiding actual reservoir operation. FELCC is the minimum generation, uniformly shaped each period (half a month) similar to the hydro-power system firm load, that the hydro-power system can produce during the historical streamflow periods on record, while optimumly drafting the available reservoir storage from full to empty. The CP is the specific period during the historical streamflow record that the hydro-power system produces the FELCC. Some of the important CP assumptions are:

1. All PNCA reservoirs must be full at the beginning of CP unless drafting for minimum flow or flood control. Usually Dworshark and Mossyrock, storage reservoirs, need this adjustment. All PNCA reservoirs must empty at the end of CP unless drafting empty would actually cause less system average CP generation. Several small reservoirs must empty before the end of the CP. Usually Priest Lake and Coeur d'Alene Lake require this change.

2. All PNCA projects must limit their CP generation to 85 percent of full-gate flow unless a higher flow is necessary to make all storage water usable during the CP. There are several projects that must exceed 85 percent full-gate flow. In the Columbia River Treaty Assured Operating Plan (AOP) studies, the Canadian projects are also limited to 85 percent of full-gate flow.

3. Critical Rule Curve (CRC) crossovers and bubbles should be avoided if the change does not adversely affect FELCC or uniformity. CRC crossovers occur when the 2nd, 3rd or 4th year CRC is above a lower


INF$_1$(I)
X$_1$(I)
S$_1$(I)
U$_1$(I)
G$_1$(I)
INF$_2$(I)
X$_2$(I)
S$_2$(I)
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U$_8$(I)
G$_8$(I)
INF$_9$(I)
X$_9$(I)
S$_9$(I)
U$_9$(I)
G$_9$(I)

Fig. 5 A Nine Reservoir Hydro System.

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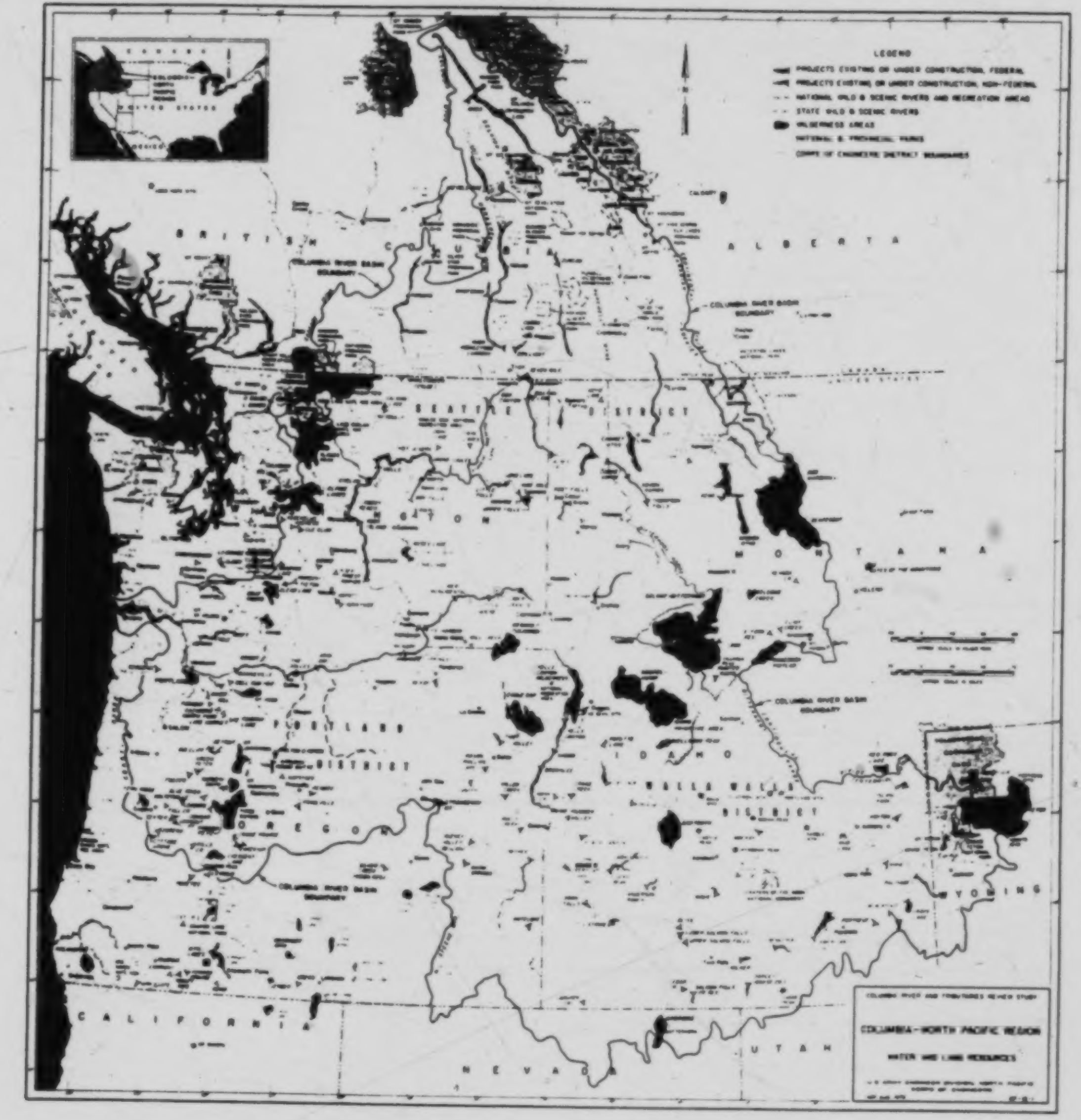
LEGEND
PROJECTS EXISTING OR UNDER CONSTRUCTION, FEDERAL
PROJECTS EXISTING OR UNDER CONSTRUCTION, NON-FEDERAL
NATIONAL WILD & SCENIC RIVERS AND RECREATION AREAS
STATE WILD & SCENIC RIVERS
WILDERNESS AREAS
NATIONAL & PROVINCIAL PARKS
CORPS OF ENGINEERS DISTRICT BOUNDARIES
CANADA
UNITED STATES
MEXICO
BRITISH COLUMBIA
ALBERTA
COLUMBIA RIVER BASIN BOUNDARY
UNITED STATES
SEATTLE DISTRICT
WASHINGTON
MONTANA
PORTLAND DISTRICT
IDAHO
WALLA WALLA DISTRICT
OREGON
WYOMING
CALIFORNIA
NEVADA
UTAH
COLUMBIA RIVER AND TRIBUTARIES REVIEW STUDY
COLUMBIA–NORTH PACIFIC REGION
WATER AND LAND RESOURCES
U.S. ARMY ENGINEER DIVISION, NORTH PACIFIC
CORPS OF ENGINEERS

Fig. 6 Map of Pacific Northwest River System.

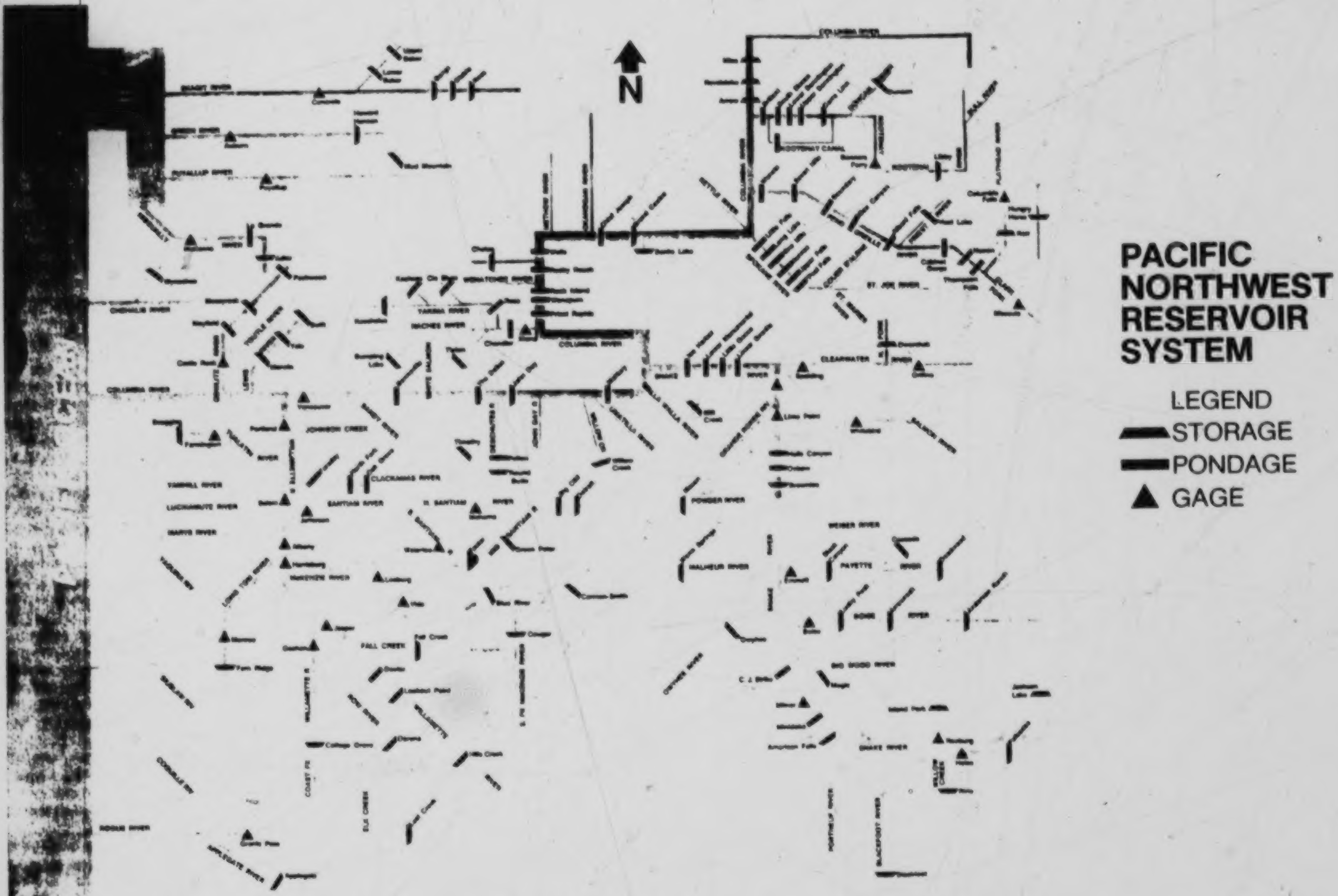
PACIFIC NORTHWEST RESERVOIR SYSTEM
LEGEND
STORAGE
PONDAGE
GAGE
N
COLUMBIA RIVER
METHOW RIVER
OKANOGAN RIVER
WENATCHEE RIVER
YAKIMA RIVER
NACHES RIVER
KOOTENAY CANAL
FLATHEAD RIVER
ST. JOE RIVER
CLEARWATER
JOHN DAY R.
DESCHUTES
POWDER RIVER
WEISER RIVER
MALHEUR RIVER
PAYETTE
BOISE
RIVER
BIG WOOD RIVER
SNAKE RIVER
PORTNEUF RIVER
BLACKFOOT RIVER
WILLOW CREEK
C. J. Strike
American Falls
JOHNSON CREEK
CLACKAMAS RIVER
SANTIAM RIVER
N. SANTIAM
YAMHILL RIVER
LUCKIAMUTE RIVER
MARYS RIVER
WILLAMETTE R.
FALL CREEK
COAST FK
ELK CREEK
S. FK McKENZIE RIVER
CHEHALIS RIVER
PUYALLUP RIVER
ROGUE RIVER
APPLEGATE RIVER

Fig. 7 Schematic Map of Pacific Northwest Reservoir System.

numbered CRC during any period. CRC bubbles occur when any project reverses a general seasonal trend of either drafting or filling. An example of a CRC bubble would be a project filling slightly during December when the general trend is a steady draft of all projects from September through April. Both crossovers and bubbles may indeed be optimum for the CP, but appear illogical for guiding a 40-year regulation. Neither crossovers nor bubbles are prohibited by the PNCA; however, when only minor violations occur, they should be fixed at the major reservoirs if it does not significantly decrease FELCC or cause uniformity problems.

4. Some of the nonmajor reservoirs do not have a significant ability to optimize total system operation and they are usually more tightly constrained by at-site nonpower constraints.

5. All project nonpower constraints must be met to the maximum extent possible. These include flood control, at-site and downstream minimum and maximum outflow constraints (including the Water Budget), spills and bypass flows, reservoir filling and drafting rate limits, and minimum and maximum elevations. Violation of any of these constraints in order to increase FELCC is almost never allowed.

[illegible]2 Critical Period Optimizer

The CP Optimzier is used to help determine the operation that maximizes the average CP generation, shaped uniformly to the load, while meeting hard constraints and balancing the violations of soft constraints. The model does this by varying the storage contents of projects, all periods, so as to maximize the objective function. The objective function is a summation of many linear and nonlinear equations, with many limiting inequalities. The model used here is functional analysis and the minimum norm formulation technique to search for the change in storage values for each project, each period (half a month), that will increase the objective function. The optimizer has to check the main CP regulation problems which are:

1. All storage projects must start the CP full and end the CP empty unless constrained by minimum flow.
2. All storage projects must not violate minimum and maximum storage constraints. Minimum and maximum outflow constraints can cause storage constraints to be violated and this should be taken care of during the planning process.
3. All storage projects' generation should be less than 85 percent full-gate flow except Kerr in June.
4. Assure that each project is regulated so that all minimum and maximum flow constraints are met. Project minimum flows are not met if a project is empty and again this should be taken care of during the planning process.
5. Water Budget minimum flow is exactly met. The Pacific Northwest Power Planning Council has established a Water Budget to help the downstream migration of fish on the Columbia and Snake Rivers. The Corps of Engineers have in turn established minimum flow constraints in order to meet the Council's Water Budget. The Water Budget minimum flows are measured at Priest Rapids Dam on the Columbia and Lower Granite Dam on the Snake River. In addition to the Water Budget, these projects also have other minimum flow constraints.

6. CRC bubbles should be fixed if no significant loss of FELCC will result.

7. CRC crossovers should be fixed if no significant loss of FELCC will result.

8. Different irrigation depletions, plant data changes, and major reservoir operation changes are the usual cause of changes in CP generation. These differences in CP average generation for each project and system total compared to similar previous studies should be explainable.

9. The PNCA requires that no reservoir draft below its Energy Content Curves (ECC) except for nonpower purposes, unless a total system draft below ECC is required to produce the system FELCC. ECC in a 40-year regulation is to determine the amount each reservoir may draft to produce nonfirm generation.

10. The optimizer should supply the B.P.A. at the end of optimization process with the new rule curves (curves that specify reservoir water levels as a funciton of time) which should guarantee a maximum generation and satisfy all hard constraints. This implies that the benefit from the recommended rule curves should be more than that of the supplied initial estimate if it is not the global maximum.

3.3 B.P.A. Hydro-Power Projects

The B.P.A. system consists of 37 storage projects and 51 run-of-river projects Fig. 7. Figure 8 presents an explanatory, schematic diagram of the whole system, the 88 projects.

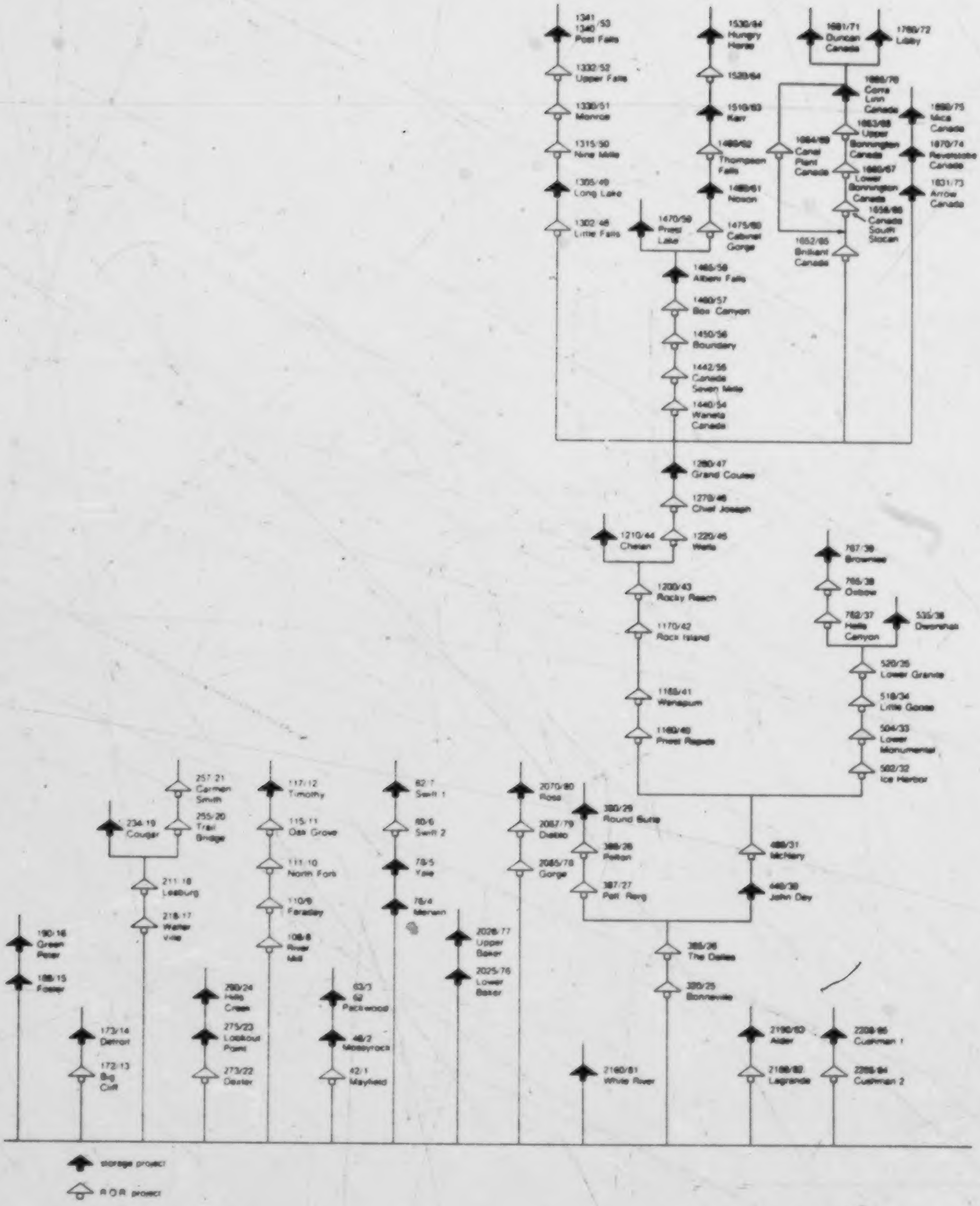
1341
1340/53
Post Falls
1332/52
Upper Falls
1330/51
Monroe
1315/50
Nine Mile
1305/49
Long Lake
1302/48
Little Falls
1530/64
Hungry Horse
1520/64
1510/63
Kerr
1490/62
Thompson Falls
1480/61
Noxon
1475/60
Cabinet Gorge
1470/59
Priest Lake
1465/58
Albeni Falls
1460/57
Box Canyon
1450/56
Boundary
1442/56
Canada
Seven Mile
1440/54
Waneta
Canada
1681/71
Duncan
Canada
1760/72
Libby
1665/70
Corra
Linn
Canada
1663/68
Upper
Bonnington
Canada
1660/67
Lower
Bonnington
Canada
1656/66
Canada
South
Slocan
1664/69
Canal
Plant
Canada
1652/65
Brilliant
Canada
1890/75
Mica
Canada
1870/74
Revelstoke
Canada
1831/73
Arrow
Canada
1280/47
Grand Coulee
1270/46
Chief Joseph
1210/44
Chelan
1220/45
Wells
1200/43
Rocky Reach
1170/42
Rock Island
1165/41
Wanapum
1160/40
Priest Rapids
767/39
Brownlee
765/38
Oxbow
762/37
Hells
Canyon
535/36
Dworshak
520/35
Lower Granite
518/34
Little Goose
504/33
Lower
Monumental
502/32
Ice Harbor
488/31
McNary
440/30
John Day
390/29
Round Butte
388/28
Pelton
387/27
Pelt. Rereg
365/26
The Dalles
320/25
Bonneville
2070/80
Ross
2067/79
Diablo
2065/78
Gorge
2028/77
Upper
Baker
2025/76
Lower
Baker
2160/81
White River
2190/83
Alder
2188/82
Lagrande
2209/85
Cushman 1
2205/84
Cushman 2
190/16
Green
Peter
188/15
Foster
173/14
Detroit
172/13
Big
Cliff
257/21
Carmen
Smith
255/20
Trail
Bridge
234/19
Cougar
211/18
Leaburg
218/17
Walter
ville
290/24
Hills
Creek
275/23
Lookout
Point
273/22
Dexter
117/12
Timothy
115/11
Oak Grove
111/10
North Fork
110/9
Faraday
108/8
River
Mill
63/3
62
Packwood
48/2
Mossyrock
42/1
Mayfield
82/7
Swift 1
80/6
Swift 2
78/5
Yale
76/4
Merwin
storage project
R.O.R. project

Fig. 8 Explanatory Schematic Diagram of the BPA Hydro Power System.

Following is a list of the 88 projects and a brief description of each one from the nonpower constraints viewpoint; tailwater tables for the storage projects; and the water to energy tables.

3.3.1 Mayfield(42)

-Run-of-river (ROR) project.

-85% Full gate flow constraints (FGFC)=12 389.6 CFS (cubic feet per second).

- Generation vs. Outflow Through the Turbine for the Mayfield Dam (Ref. 74, Table 1)

- Minimum Flow Requirements for the Mayfield Dam (Ref. 74, Table 2)

3.3.2 Mossyrock (48)

- Mossyrock Dam is the major storage project for a series of dams downstream on the Cowlitz River located at river mile.

- Project purposes include power generation, flood control, and power storage.

- It has an average water conversion factor (WCF) of 22.26 KW/CFS.

- No monthly limits on maximum draft rate.

- Maximum storage = 654.3 KSFD (KSFD=$86.4*10^6 ft^3$)

- Minimum storage = 0.0

- Maximum elevation = 778.5 ft

- Minimum elevation = 621.5 ft

- FGFC = 13 370 CFS

- Minimum flow = 1000 CFS

- Tailwater Elevation (TWE) vs. Total Outflow (Outflow Through the Turbine Plus the Spill) for the Mossyrock Dam (Ref. 74, Table 3)

- Storage vs. Forebay Elevation for the Mossyrock Dam (Ref. 74, Table 4)

- Head vs. Water Conversion Factor (Watt/CFS) for the Mossyrock Dam (Ref. 74, Table 5)

3.3.3 **Packwood (62-63)**

- Two projects one is a ROR and the other is a storage project.

- 85% FGFC = 233.7 CFS
- Minimum outflow = 10.0 CFS
- Maximum storage = 1.4 KSFD
- Minimum storage = 0.0

- Total Generation vs. Outflow Through the Turbine for the Packwood Dam (Ref. 74, Table 6)

3.3.4 **Merwin (76)**

- Storage project
- Maximum storage = 92.1 KSFD
- Minimum storage = 34.3 KSFD
- Maximum outflow = 11400 CFS

- Minimum Outflow Requirements for the Merwin Dam (Ref. 74, Table 7)

- Storage vs. Forebay Elevation for Merwin Dam (Ref. 74, Table 8)

- Tailwater Elevation vs. Total Outflow for the Merwin Dam (Ref. 74, Table 9)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Merwin Dam (Ref. 74, Table 10)

3.3.5 **Yale (78)**

- Storage project
- Maximum storage = 95.6 KSFD
- Minimum storage = 0.0
- Maximum outflow = 8000 CFS
- Minimum outflow = 0.0

- Storage vs. Forebay Elevation for the Yale Dam (Ref. 74, Table 11)

- Tailwater Elevation vs. Total Outflow for the Yale Dam (Ref. 74, Table 12)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Yale Dam (Ref. 74, Table 13)

3.3.6 Swift II (80)

- ROR project
- Maximum outflow = 8,600 CFS
- Minimum outflow = 0.0
- Forebay elevation = 603.0 feet

- Tailwater Elevation vs. total Outflow for the Swift II Dam (Ref. 74, Table 14)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Swift II Dam (Ref. 74, Table 15)

3.3.7 Swift I (82)

- Storage project
- Maximum storage = 225.4 KSFD
- Minimum storage = 0.0
- Maximum outflow = 935.0 CFS
- Minimum outflow = 0.0
- Tailwater elevation = 605.0 feet

- Storage vs. Forebay Elevation for the Swift I Dam (Ref. 74, Table 16)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Swift I Dam (Ref. 74, Table 17)

3.3.8 River Mill (108)

- ROR project
- 85% FGFC = 3 833.5 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the River Mill Dam (Ref. 74, Table 18)

3.3.9 Faraday (110)

- ROR project
- 85% FGFC = 4 109.7 CFS
- Minimum flow = 0.0

- Generation vs. Outflow Through the Turbine for the Faraday Dam (Ref. 74, Table 19)

3.3.10 North Fork (111)

- ROR project
- 85% FGFC = 4 636.7 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the North Fork Dam (Ref. 74, Table 20)

3.3.11 Oak Grove (115)

- ROR project
- 85% FGFC = 454.7 CFS

- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the Oak Grove Dam (Ref. 74, Table 21)

3.3.12 **Timothy (117)**

- Storage project
- Maximum storage = 31.1 KSFD
- Minimum storage = 0.0
- Maximum outflow = 535.0 CFS
- Minimum outflow = 40.0 CFS
- No generation

3.3.13 **Bill Cliff (172)**

- ROR project
- 85% FGFC = 3 060.0 CFS

- Minimum Flow Requirement for the Big Cliff Dam (Ref. 74, Table 22)

- Generation vs. Outflow Through the Turbine for the Big Cliff Dam (Ref. 74, Table 23)

3.3.14 **Detroit (173)**

- Storage project
- Maximum storage = 162.0 KSFD
- Minimum storage = 0.0
- Maximum outflow = 5,340 CFS
- Tailwater elevation = 1,202 feet

- Minimum Flow Requirement for the Detroit Dam (Ref. 74, Table 24)

- Generation vs. Outflow Through the Turbine for the Detroit Dam (Ref. 74, Table 25)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Detroit Dam (Ref. 74, Table 26)

3.3.15 **Foster (188)**

- Storage project
- Maximum storage = 14.3 KSFD
- Minimum storage = 0.0
- Maximum outflow = 3 200.0 CFS
- Tailwater elevation = 527.0 feet

- Minimum Flow Requirement for the Foster Dam (Ref. 74, Table 27)

- Storage vs. Forebay Elevation for the Foster Dam (Ref. 74, Table 28)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Foster Dam (Ref. 74, Table 29)

3.3.16 **Green Peter (190)**

- Storage project
- Maximum storage = 157.8 KSFD
- Minimum storage = 0.0

- Maximum outflow = 4,600 CFS
- Minimum outflow = 50 CFS
- Tailwater elevation = 700.0 feet

- Storage vs. Forebay Elevation for the Green Peter Dam (Ref. 74, Table 30)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Green Peter Dam (Ref. 74, Table 31)

3.3.17 **Walterville (218)**

- ROR project
- 85% FGFC = 2,193.0 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the Walterville Dam (Ref. 74, Table 32)

3.3.18 **Leaburg (221)**

- ROR project
- 85% FGFC = 2,159.0 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the Leaburg Dam (Ref. 74, Table 33)

3.3.19 **Cougar (234)**

- Storage project
- Maximum storage = 77.4 KSFD
- Minimum storage = 0.0
- Maximum outflow = 2,100.0 CFS
- Tailwater elevation = 1,254.0 feet

- Minimum Flow Requirements for the Cougar Dam (Ref. 74, Table 34)

- Storage vs. Forebay Elevation for the Cougar Dam (Ref. 74, Table 35)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Cougar Dam (Ref. 74, Table 36)

3.3.20 **Trail Bridge (255)**

- ROR project
- 85% FGFC = 578.0 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the Trail Bridge Dam (Ref. 74, Table 37)

3.3.21 **Carmen Smith (257)**

- ROR project
- 85% FGFC = 382.5 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the Carmen Smith Dam (Ref. 74, Table 38)

3.3.22 **Dexter (273)**

- ROR project
- 85% FGFC = 3,697.5 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the Dexter Dam (Ref. 74, Table 39)

3.3.23 **Lookout Point (275)**

- Storage project
- Maximum storage = 169.7 KSFD
- Minimum storage = 0.0
- Maximum outflow = 9,300.0 CFS
- Tailwater elevation = 6,930 feet

- Minimum Flow Requirement for the Lookout Point Dam (Ref. 74, Table 40)

- Storage vs. Forebay Elevation for the Lookout Point Dam (Ref. 74, Table 41)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Lookout Point Dam (Ref. 74, Table 42)

3.2.24 Hills Creek (290)

- Storage project
- Maximum storage = 122.8 KSFD
- Minimum storage = 0.0
- Maximum outflow = 1,800.0 CFS
- Minimum outflow = 100.0 CFS
- Tailwater elevation = 1,226.0 feet

- Storage vs. Forebay Elevation for the Hills Creek Dam (Ref. 74, Table 43)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Hills Creek Dam (Ref. 74, Table 44)

3.3.25 Bonneville (320)

- ROR project
- 85% FGFC = 262,225.0 CFS
- Minimum outflow = 10,000.0 CFS

- Generation vs. Outflow Through the Turbine for the Bonneville Dam (Ref. 74, Table 45)

3.3.26 Dalles (365)

- ROR project
- 85% FGFC = 318,750.0 CFS

- Minimum Flow Requirement for the Dalles Dam (Ref. 74, Table 46)

- Generation vs. Outflow Through the Turbine for the Dalles Dam (Ref. 74, Table 47)

3.3.27 **Pelt. Rereg (387)**

- ROR project
- 85% FGFC = 5,440.0 CFS

- Minimum Flow Requirement for the Pelt. Rereg Dam (Ref. 74, Table 48)

- Generation vs. Outflow Through the Turbine For the Pelt. Rereg Dam (Ref. 74, Table 49)

3.3.28 **Pelton (388)**

- ROR project
- 85% FGFC = 9,804.7 CFS
- Minimum outflow = 3,000.0 CFS

- Generation vs. Outflow Through the Turbine for the Pelton Dam (Ref. 74, Table 50)

3.2.29 **Round Butte (390)**

- Storage project
- Maximum storage = 138.26 KSFD

- Minimum storage = 0.0
- Maximum outflow = 11,200.0 CFS
- Minimum outflow = 0.0
- TWE = 1,578.0 feet

- Storage vs. Forebay Elevation for the Round Butte Dam (Ref. 74, Table 51)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Round Butte Dam (Ref. 74, Table 52)

3.3.30 **John Day (440)**

- Storage project
- Maximum storage = 269.7 KSFD
- Minimum storage = 0.0
- Maximum outflow = 322,000.0 CFS

- Minimum Flow Requirement for the John Day Dam (Ref. 74, Table 53)

- Storage vs. Forebay Elevation for the John Day Dam (Ref. 74, Table 54)

- Tailwater Elevation vs. Total Outflow for the John Day Dam (Ref. 74, Table 55)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for John Day Dam (Ref. 74, Table 56)

3.3.31 **McNary (488)**

- ROR project
- Maximum outflow = 202,000 CFS

- Minimum Flow Requirements for the McNary Dam (Ref. 74, Table 57)

- Tailwater vs. Total Outflow for the McNary Dam (Ref. 74, Table 58)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the McNary Dam (Ref. 74, Table 59)

3.3.32 **Ice Harbor (502)**

- ROR project
- 85% FGFC = 89,250.0 CFS

- Minimum Flow Requirements for the Ice Harbor Dam (Ref. 74, Table 60)

- Generation vs. Outflow Through the Turbine for the Ice Harbor Dam (Ref. 74, Table 61)

3.3.33 **Lower Monumental (504)**

- ROR project

- 85% FGFC = 113,900.0 CFS

- Minimum Flow Requirements for the Lower Monumental Dam (Ref. 74, Table 61)

- Generation vs. Outflow Through the turbine for the Lower Monumental Dam (Ref. 74, Table 63)

3.3.34 **Little Goose (518)**

- ROR project
- 85% FGFC = 113,900.0 CFS

- Minimum Flow Requirements for the Little Goose Dam (Ref. 74, Table 64)

- Generation vs. Outflow Through the Turbine for the Little Goose Dam (Ref. 74, Table 65)

3.3.35 **Lower Granite (520)**

- ROR project
- 85% FGFC = 113,900.0

- Minimum Flow Requirements for the Granite Dam to Assure the Water Budget Minimum Flows on the Snake River and Other Minimum Flow Constraints (Ref. 74, Table 66)

- Generation vs. outflow Through the Turbine for the Lower Granite Dam (Ref. 74, Table 67)

3.3.36 **Dworshak (535)**

- Storage project

- Dworshak was constructed for power, flood control, and navigation. Fishery and recreation uses are not project authorized uses, but are allowed at the Corps' discretion.

- Maximum storage = 1,016.0 KSFD
- Minimum storage = 0.0
- Maximum forebay elevation = 1,600.0 feet
- Minimum forebay elevation = 1,445.0 feet
- Maximum outflow = 10,000.0 CFS
- Minimum outflow = 1,000.0 CFS

- Storage vs. Forebay Elevation for the Dworshak Dam (Ref. 74, Table 68)

- Tailwater Elevation vs. Total Outflow for the Dwoshak Dam (Ref. 74, Table 69)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Dworshak Dam (Ref. 74, Table 70)

3.3.37 **Hells Canyon (762)**

- ROR project

- 85% FGFC = 25,840.0 CFS
- Minimum outflow = 5,000 CFS

- Generation vs. Outflow Through the Turbine for the Hells Canyon Dam (Ref. 74, Table 71)

3.3.38 **Oxbow (765)**

- ROR project
- 85% FGFC = 23,970.0 CFS
- Minimum outflow = 5,000.0 CFS

- Generation vs. Outflow Through the Turbine for the Oxbow Dam (Ref. 74, Table 72)

3.3.39 **Brownlee (767)**

- Storage project
- Maximum storage = 491.7 KSFD
- Minimum storage = 0.6 KSFD
- Maximum outflow = 34,500.0 CFS

- Minimum Flow Requirement for Brownlee Dam (Ref. 74, Table 73)

  - Minimum forebay elevation = 1,976 feet

- Maximum Forebay Elevation for the Brownlee Dam (Ref. 74, Table 74)

- Maximum Draft Rate for the Brownlee Dam (Ref. 74, Table 75)

- Storage vs. Forebay Elevation for the Brownlee Dam (Ref. 74, Table 76)

    - Tailwater elevation = 1,802.5 feet

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Brownlee Dam (Ref. 74, Table 77)

3.3.40 **Priest Rapids (1160)**

- ROR project
- 85% FGFC = 148,750.0 CFS

- Minimum Flow Requirement for the Priest Rapids Dam to Assure the Water Budget Minimum Flows on the Columbia River and Other Minimum Flow Constraints (Ref. 74, Table 778)

- Because Priest Rapids or the other four mid-Columbia projects do not have seasonal storage, the regulations require that the Water Budget (WB) minimum flow constraints be transferred upstream. The May WB minimum flow requirement at Priest Rapids is so large that a normal at-site minimum flow requirement at Grand Coulee would frequently cause the project to draft empty. This is because the model satisfies at-site flow constraints before determining drafting rights for system load or flow needs. The problem then is to determine which upstream projects should draft and in what proportion they should draft for WB minimum flows.

- Generation vs. Outflow Through the Turbine for the Priest Rapids Dam (Ref. 74, Table 79)

3.3.41 **Wanapum (1165)**

- ROR project
- 85% FGFC = 147,900.0 CFS
- Minimum outflow = 36,000.0 CFS

- Generation vs. Outflow Through the Turbine for the Wanapum Dam (Ref. 74, Table 80)

3.3.42 **Rock Island (1170)**

- ROR project
- 85% FGFC = 182,750.0 CFS
- Minimum outflow = 35,000.0 CFS

- Generation vs. Outflow Through the Turbine for the Rock Island Dam (Ref. 74, Table 81)

3.3.43 **Rocky Reach (1200)**

- ROR project
- 85% FGFC = 187,000.0 CFS
- Minimum outflow = 30,000.0 CFS

- Generation vs. Outflow Through the Turbine for the Rocky Reach Dam (Ref. 74, Table 82)

3.3.44 Chelan (1210)

- Storage project
- Low head dam on Chelan River to control the outflows from

Lake Chelan, a large natural lake.

- Maximum storage = 341.5 KSFD
- Minimum storage = 0.0
- Maximum elevation = 1100.0 feet
- Minimum elevation = 1079.0 feet
- Maximum flow = 2,016.0 CFS
- Minimum flow = 50.0 CFS
- Tailwater elevation = 707.0 feet

- Storage vs. Forebay Elevation for the Chelan Dam (Ref. 74, Table 83)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Chelan Dam (Ref. 74, Table 84)

3.3.45 Wells (1220)

- ROR project
- 85% FGFC = 212,500.0 CFS
- Minimum flow = 30,000.0 CFS

- Generation vs. Outflow Through the Turbine for the Wells Dam (Ref. 74, Table 85)

3.3.46 Chief Joseph (1270)

- ROR project
- Maximum outflow = 215,000 CFS
- Minimum outflow = 30,000 CFS
- Forebay elevation = 956.0 feet

- Tailwater Elevation vs. Total Outflow for the Chief Joseph Dam (Ref. 74, Table 86)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Chief Joseph Dam (Ref. 74, Table 87)

3.3.47 Grand Coulee (1280)

- Storage project

- The largest reservoir, dam and powerhouse in the U.S. portion of the Columbia River Basin. Situated at the top of a string of large ROR projects. It is the single most important project of the U.S. Federal Columbia River Power System. It has the largest amount of control over monthly and weekly river operations. Water released from Grand Coulee flows through 10 downstream projects with each KSFD producing a total of about 2.8 MW months of energy. On a smaller scale, 10 cubic feet of water (about 75 gallons) released from Grand Coulee will produce 240 watt hours of energy, worth about 1 cent to the residential consumer.

- Maximum storage = 2,614.3 KSFD
- Minimum storage = 0.0
- Maximum elevation = 1,290.0 feet
- Minimum elevation = 1 208.0 feet
- Irrigation, flood control, minimum flows, and power impacts are the major criteria for developing constraints on reservoir operation. Recreation, navigation, and at-site fish needs, do not significantly affect reservoir operation.
- Maximum outflow = 280,000.0 CFS
- The WB minimum flow requirements at Priest Rapids Dam are so large that a normal at-site minimum flow requirement would frequently cause Grand Coulee to draft empty. This is because the model satisfies at-site flow constraints before determining drafting rights for system load or flow needs.

- Minimum Flow Requirement for the Grand Coulee Dam to Assure the Water Budget Minimum Flows on the Columbia River and Other Minimum Flow Constraints (Ref. 74, Table 88)

- Storage vs. Forebay Elevation for the Grand Coulee Dam (Ref. 74, Table 89)

- Tailwawter Elevation vs. Total Outflow for the Grand Coulee Dam (Ref. 74, Table 90)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Grand Coulee Dam (Ref. 74, Table 91)

3.3.48 **Little Falls** (1280)

- ROR project
- 85% FGFC = 5,950.0 CFS
- Minimum flow = 0.0

- Generation vs. Outflow Through the Turbine for the Little Falls Dam (Ref. 74, Table 92)

3.3.49 **Long Lake** (1305)

- Storage project
- Maximum storage = 52.5 KSFD
- Minimum storage = 0.0
- Maximum outflow = 6,300.0 CFS
- Minimum outflow = 0.0
- Tailwater elevation = 1,363.0 feet

- Storage vs. Forebay Elevation for the Long Lake Dam (Ref. 74, Table 93)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Long Lake Dam (Ref. 74, Table 94)

3.3.50 **Nine Mile (1315)**

- ROR project
- 85% FGFC = 3 910.0 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the turbine for the Nine Mile Dam (Ref. 74, Table 95)

3.3.51 **Monroe Street (1330)**

- ROR project
- 85% FGFC = 2,040.0 CFS
- Minimum flow = 0.0

- Generation vs. Outflow Through the turbine for the Monroe Street Dam (Ref. 74, Table 96)

3.3.52 **Upper Falls (1332)**

- ROR project
- 85% FGFC = 2,125.0 CFS
- Minimum flow = 300.0 CFS

- Generation vs. Outflow Through the Turbine for the Upper Falls Dam (Ref. 74, Table 97)

3.3.53 **Post Falls and Lake Cour D'Alene (1341-1340)**

- Two projects, one is a ROR project and the other is a storage project.

- Maximum storage = 112.5 KSFD
- Minimum storage = 0.0
- 85% FGFC = 3,995.0 CFS
- Minimum flow = 300.0 CFS

- Generation vs. Outflow Through the Turbine for the Post Falls Dam (Ref. 74, Table 98)

3.3.54 **Waneta (1440)**

- ROR project on the Canadian side of the border.
- 85% FGFC = 21,250.0 CFS
- Minimum flow = 0.0

- Generation vs. Outflow Through the Turbine for the Waneta Dam (Ref. 74, Table 99)

3.3.55 **Seven Mile (1442)**

- ROR project on the Canadian side of the border.
- 85% FGFC = 31,025.0 CFS
- Minimum flow = 0.0

- Generation vs. outflow Through the Turbine for the Seven Mile Dam (Ref. 74, Table 100)

3.3.56 **Boundary (1450)**

- ROR project

- 85% FGFC = 45,645.0 CFS
- Minimum flow = 0.0

- Generation vs. Outflow Through the Turbine for the Boundary Dam (Ref. 74, Table 101)

3.3.57 **Box Canyon (1460)**

- ROR project
- 85% FGFC = 24,820.0 CFS
- Minimum flow = 0.0

- Generation vs. Outflow Through the Turbine for the Box Canyon Dam (Ref. 74, Table 102)

3.3.58 **Albeni Falls (1465)**

- Storage project

- Low head dam on the Pend Oreille river downstream from Pend Oreille Lake, a natural lake. The project controls outflows from Pend Oreille Lake for flood control, navigation, power, recreation, and fish and wildlife conservation objectives.

- Maximum hydraulic capacity = 33,000 CFS
- Bypass flow = 50 CFS

- Maximum Storage Requirements for the Albeni Falls Dam (Ref. 74, Table 103)

- Minimum Storage Requirement for the Albeni Falls Dam (Ref. 74, Table 104)

- Maximum Elevation Requirement for the Albeni Falls Dam (Ref. 74, Table 105)

- Minimum Elevation Requirement for the Albeni Falls Dam (Ref. 74, Table 106)
    - Minimum flow requirement = 4,000 CFS

- Maximum Draft Rate Constraints for the Albeni Falls Dam (Ref. 74, Table 107)

- Storage vs. Forebay Elevation for the Albeni Falls Dam (Ref. 74, Table 108)

- Tailwater Elevation vs. Total Outflow for the Albeni Falls Dam (Ref. 74, Table 109)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Albeni Falls Dam (Ref. 74, Table 110)

3.3.59 **Priest Lake (1470)**

- Storage project
- Maximum storage constraint = 35.5 KSFD
- Minimum storage constraint = 0.0
- Maximum outflow = 2,820 CFS
- Minimum outflow = 60 CFS
- No generation

3.3.60 **Cabinet Gorge (1475)**

- ROR project
- 85% FGFC = 30,090.0 CFS
- Minimum flow = 3 000.0 CFS

- Generation vs. Outflow Through the Turbine for the Cabinet Gorge Dam (Ref. 74, Table 111)

3.3.61 **Noxon (1480)**

- Storage project
- Maximum storage constraint = 116.3 KSFD
- Minimum storage constraint = 0.0
- Maximum outflow = 46,900 CFS
- Minimum outflow = 0.0

- Storage vs. Forebay Elevation for the Noxon Dam (Ref. 74, Table 112)

- Tailwater Elevation vs. Total Outflow for the Noxon Dam (Ref. 74, Table 113)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Noxon Dam (Ref. 74, Table 114)

3.3.62 Thompson Falls (1490)

- ROR project
- 85% FGFC = 9,435.0 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the Thompson Falls Dam (Ref. 74, Table 115)

3.3.63 Kerr (1510)

- Storage project
- Low head dam downstream of Flathead Lake on the Flathead River.
- Maximum hydraulic capacity = 14,346 CFS
- Kerr always has a 85 percent fall-gate violation during June.
- Maximum storage = 614.7 KSFD
- Minimum storage = 0.0
- Maximum elevation = 2,893.0 feet
- Minimum elevation = 2,883.0 feet
- Maximum outflow = 14,346.0 CFS
- Minimum outflow = 3,200.0 CFS
- Tailwater elevation = 2,705.0 feet

- Storage vs. Forebay Elevation for the Keer Dam (Ref. 74, Table 116)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Kerr Dam (Ref. 74, Table 117)

3.3.64 Colfls and Hungry Horse (1520-1530)

a) **Colfls Dam (1520)**

- It has neither storage capacity nor power house generation.

b) **Hungry Horse (1530)**

- Has the msot valuable storage in the Federal Columbia River Power System with an average CP system water conversion factor of 155 MW/KCFS.

- Maximum hydraulic capacity varies with head, but it is 8900 CFS at critical head. Critical head is the minimum head that maximum generation can be maintained.

- Maximum Flow Constraints for the Hungry Horse Dam (Ref. 74, Table 118)

  - Minimum flow requirement = 400 CFS
  - Maximum storage = 1,593.6 KSFD
  - Minimum storage = 0.0
  - Maximum elevation = 3,560.0 feet
  - Minimum elevation = 3,336.0 feet

- Storage vs. Forebay Elevation for the Hungry Horse Dam (Ref. 74, Table 119)

- Tailwater Elevation vs. Total Outflow for the Hungry Horse Dam (Ref. 74, Table 120)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Hungry Horse Dam (Ref. 74, Table 121)

3.3.65 **Brilliant (1652)**

- ROR project on the Canadian side of the border
- 85% FGFC = 15,300.0 CFS
- Minimum flow = 3,500.0 CFS

- Generation vs. Outflow Through the Turbine for the Brilliant Dam (Ref. 74, Table 122)

3.3.66 **South Slocan (1658)**

- ROR project on the Canadian side of the border
- 85% FGFC = 9,171.5 CFS
- Minimum flow = 3,500.0 CFS

- Generation vs. Outflow Through the Turbine for the South Slocan Dam (Ref. 74, Table 123)

3.3.67 **Lower Bonnington (1660)**

- ROR project on the Canadian side of the border
- 85% FGFC = 7,820.0 CFS
- Minimum flow = 3,500.0 CFS

- Generation vs. Outflow Through the Turbine for the Lower Bonnington Dam (Ref. 74, Table 124)

3.3.68 **Upper Bonnington (1663)**

- ROR project on the Canadian side of the border
- 85% FGFC = 11,322.0 CFS
- Minimum flow = 3,500.0 CFS

- Generation vs. outflow Through the Turbine for the Upper Bonnington Dam (Ref. 74, Table 125)

3.3.69 **Canal Plant (1664)**

- ROR project on the Canadian side of the border
- 85% FGFC = 22,950.0 CFS
- Minimum flow = 0.0
- Starts at the Corra Linn Dam and discharges flow back into the Kootenay river upstream of the Brilliant plant, bypassing the Upper and Lower Bonnington and South Slocan plants Fig. 9.

- Distribution of Water Budget Between the Corra Linn Dam and the Canal Plant Dam (Ref. 74, Table 126)

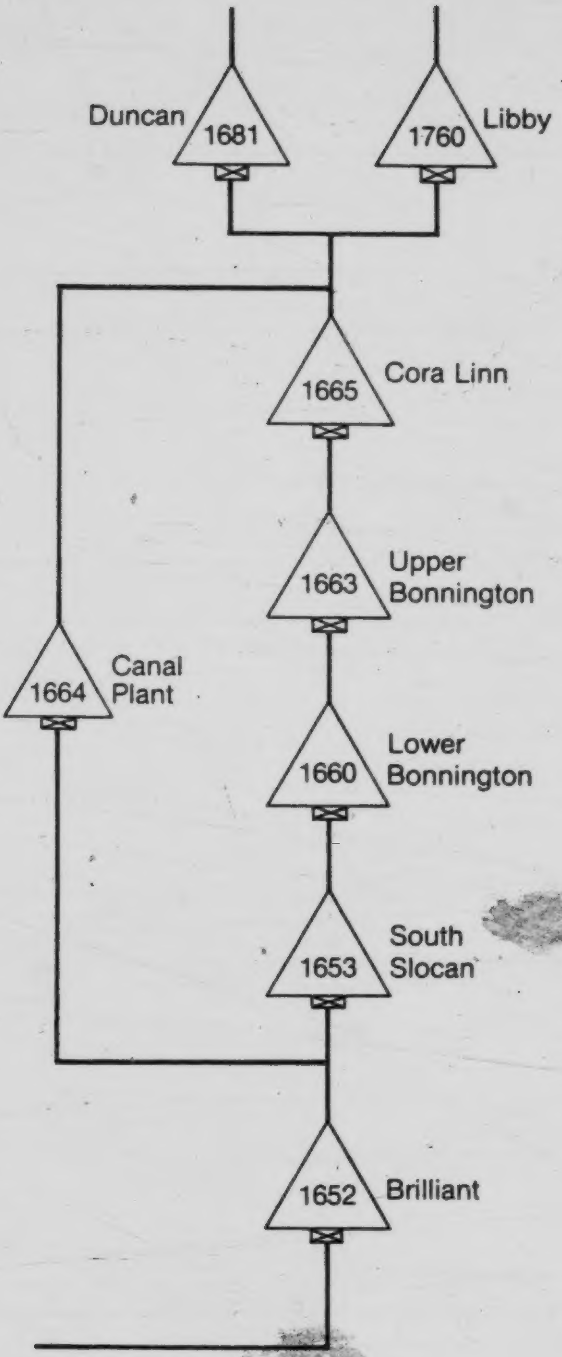
Duncan
1681
1760
Libby
1665
Cora Linn
1663
Upper Bonnington
1664
Canal Plant
1660
Lower Bonnington
1653
South Slocan
1652
Brilliant

Fig. 9 Explanatory Schematic Diagram of the Canal Plant Project.

- Generation vs. Outflow Through the Turbine for the Canal Plant Dam (Ref. 74, Table 127)

3.3.70 Corra Linn (1665)

- Storage project on the Canadian side of the border
- Lies on the Kootenay River downstream of Kootenay Lake
- Corra Linn outflows in excess of 500 CFS and up to 32000 CFS are diverted from the Corra Linn powerhouse to the more efficient Canal Plant which has a 528 MW powerhouse and a full-gate capacity of 2700 CFS. The Canal Plant discharges flow back into the Kootenay river upstream of the Brilliant Plant, bypassing the Upper and Lower Bonnington and South Slocan plants, Fig 9. Corra Linn inflows in excess of 32000 CFS are passed through Corra Linn, Table 125. The hydro models are hardwired for this operation.

- Maximum Storage Requirement for the Corra Linn Dam (Ref. 74, Table 128)
  - Minimum storage = 69.8 KSFD

- Maximum Elevation Requirement for the Corra Linn Dam (Ref. 74, Table 129)
  - Minimum elevation = 1739.3 feet

- Storage vs. Maximum Outflow for the Corra Linn Dam (Ref. 74, Table 130)

- Minimum outflow = 5,000.0 CFS

- Storage vs. Forebay Elevation for the Corra Linn Dam (Ref. 74, Table 131)

- Tailwater Elevation vs. Total Outflow for the Corra Linn Dam (Ref. 74, Table 132)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Corra Linn Dam (Ref. 74, Table 133)

3.3.71 **Duncan (1681)**

- Storage project on the Canadian side of the border

- Duncan is the third and smallest of the Columbia River Treaty projects in Canada. Duncan Lake was a natural lake before construction of the dam.

- There is no powerhouse at Duncan.

- Draft proportionally to meet U.S. system load.

- Operation of Duncan is limited by the International Joint Commission rules on Kootenay Lake. The major requirement on Duncan is to limit spring outflows so as not to cause Kootenay Lake to exceed elevations that would have occurred from filling on restriction without upstream storage.

- Duncan is the second most important reservoir (after Arrow) for uniforming CP generation because of its large storage capacity and no at-site generation. CRC crossovers and bubbles should be corrected when only a major change is needed to avoid them.

- Maximum storage = 705.8 KSFD
- Minimum storage = 0.0
- Maximum outflow = 20,000.0 CFS
- Minimum outflow = 100.0 CFS
- Maximum elevation = 1892.0 feet
- Minimum elevation = 1794.2 feet
- Maximum drft rate = 1.0 foot/day

- Storage vs. Forebay Elevation for the Duncan Dam (Ref. 74, Table 134)

3.3.72 Libby (1760)

- Storage project

- The primary purposes for the construction of Libby Dam were to provide flood control protection and to produce power. The major downstream operational limitation for Libby is to avoid the Corra Linn outflow channel restriction that can force Kootenay Lake to fill above the prescribed rule curves if the outflows from Libby and Duncan are too great. Downstream reservoirs include Corra Linn (in Canada), Grand Coulee, and John Day. There are no reservoirs upstream of Libby.

- Flood control, outflow restriction at Corra Linn, minimum outflows, and refill are the major constraints on reservoir operation. Navigation, and at-site fish needs do not significantly affect reservoir operation.

- Maximum storage = 2,510.5 KSFD
- Minimum storage = 0.0

- Maximum elevation = 2,459.0 feet
- Minimum elevation = 2,287.0 feet
- Maximum outflow = 24,100.0 CFS
- Minimum outflow = 2,000.0 CFS
- Bypass flow = 200.0 CFS
- Maximum draft rate = 1.8 feet/day

- Storage vs. Forebay Elevation for the Libby Dam (Ref. 74, Table 135)

- Tailwater Elevation vs. Total Outflow for the Libby Dam (Ref. 74, Table 136)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Libby Dam (Ref. 74, Table 137)

3.3.73 **Arrow (1831)**

- Storage project on the Canadian side of the border on the Columbia River.

- Does not have a powerhouse

- Treaty operating plans, irrigation, flood control, minimum flows, and power impacts are the major criteria for developing constraints on reservoir operation. However, recreation, navigation, and at-site fish needs do significantly affect reservoir operation. The Canadian entity will sometimes use non-treaty storage at Mica, Revelstoke, and Duncan to meet Arrow Treaty outflow requirements, in order to maintain Arrow reservoir at elevations desired by Canada.

- The Canadians prefer that Arrow be maintained above 1440.0 feet (3343.3 KSFD) elevation during the summer months in order to avoid severe dust problems at the town of Revelstoke. It is not yet clear whether or not this should be treated as even a soft constraint since the Canadians always have the option to use non-treaty storage to raise Arrow reservoir elevation in the summer.

- Arrow is the most important reservoir for uniforming CP generation because of its large storage capacity, location just upstream of Grand Coulee, and there is no on-site generation.

- Storage vs. Maximum Outflow for the Arrow Dam (Ref. 74, Table 138)
    - Minimum outflow = 5,000 CFS
    - Maximum storage (treaty) = 3,579.6 KSFD
    - Minimum storage = 0.0
    - Maximum draft rate = 1.0 foot/day
    - Maximum elevation (treaty) = 1,440.0 feet
    - Minimum elevation = 1,377.93 feet
    - Maximum storage content (non-treaty) = 3,711.7 KSFD
    - Maximum elevation (non-treaty) = 1,446.0 feet

- Storage vs. Forebay Elevation for the Arrow Dam (Ref. 74, Table 139)

3.3.74 **Revelstoke (1870)**

- Storage project on the Canadian side of the border.
- Maximum storage = 557.0 KSFD
- Minimum storage = 0.0

- Maximum outflow = 56,000.0 CFS
- Minimum outflow = 0.0
- Tailwater elevation = 1,457.0 feet

- Storage vs. Forebay Elevation for the Revelstoke Dam (Ref. 74, Table 140)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Revelstoke Dam (Ref. 74, Table 141)

3.3.75 Mica (1890)

- Constructed under the terms of the Columbia River Treaty. It has created the largest treaty storage reservoir (Kinbasket Lake), containing 11,953,000 acre-feet of active storage. The operation of the reservoir is governed by the Treaty Assured Operating Plan and Detailed Operating Plan. In these operating plans, Mica and Arrow (Keenleyside) are operated as if they were one reservoir. That is, Mica is regulated to provide optimum generation in Canada, and Keenleyside is operated to make up the difference between Mica operating for Canadian optimum versus U.S. optimum.

- Flood control, minimum flows, and Canadian power impacts are the major constraints on reservoir operation. Recreation, navigation, and at-site fish needs do not significantly affect reservoir operation.

- In 1984, BPA and B.C. Hydro signed a 10-year agreement allowing both parties the right to use non-treaty storage in Mica, Revelstoke, and Arrow for storing or drafting, and allowing the initial filling of Revelstoke reservoir, and to help fill treaty storage during low water years.

- Maximum storage content = 6, 073.0 KSFD
- Minimum storage (treaty) = 2,543.8 KSFD
- Minimum storage content = 0.0
- Maximum elevation = 2,475.0 feet
- Minimum elevation (treaty) = 2,400.0 feet
- Minimum elevation = 2,320.0 feet
- Maximum outflow = 40,000.0 CFS
- Minimum outflow = 3,000.0 CFS

- Storage vs. Forebay Elevation for the Mica Dam (Ref. 74, Table 142)

- Tailwater Elevation vs. Total Outflow for the Mica Dam (Ref. 74, Table 143)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Mica Dam (Ref. 74, Table 144)

3.3.76 **Lower Baker (2025)**

- Storage project
- Maximum storage = 71.8 KSFD
- Minimum storage = 25.3 KSFD
- Maximum outflow = 4,000.0 CFS
- Minimum outflow = 0.0
- Tailwater elevation = 180.0 feet

- Storage vs. Forebay Elevation for the Lower Baker Dam (Ref. 74, Table 145)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Lower Baker Dam (Ref. 74, Table 146)

3.3.77 **Upper Baker (2028)**

- Storage project
- Maximum storage = 111.2 KSFD
- Minimum storage = 18.1 KSFD
- Maximum outflow = 5,100.0 CFS
- Minimum outflow = 0.0

- Storage vs. Forebay Elevation for the Upper Baker Dam (Ref. 74, Table 147)

- Tailwater Elevation vs. Total Outflow for the Upper Baker Dam (Ref. 74, Table 148)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Upper Baker Dam (Ref. 74, Table 149)

3.3.78 **Gorge (2065)**

- ROR project
- 85% FGFC = 6,324.0 CFS

- Minimum Flow Requirement for the Gorge Dam (Ref. 74, Table 150)

- Generation vs. Outflow Through the Turbine for the Gorge Dam (Ref. 74, Table 151)

3.3.79 **Diablo (2067)**

- ROR project
- 85% FGFC = 5,538.6 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the Diablo Dam (Ref. 74, Table 152)

3.3.80 **Ross (2070)**

- Storage project
- The uppermost of a string of dams on the Skagit River, Washington, located at river mile.

- Maximum Flow Requirement for the Ross Dam (Ref. 74, Table 153)

- Minimum Flow Requirement for the Ross Dam (Ref. 74, Table 154)

  - Maximum storage = 530.5 KSFD
  - Minimum storage = 0.0
  - Maximum elevation = 1,602.5 feet
  - Minimum elevation = 1,475.0 feet

- Tailwater elevation = 1,205.0 feet

- Storage vs. Forebay Elevation for the Ross Dam (Ref. 74, Table 155)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Ross Dam (Ref. 74, Table 156)

3.3.81 **White River (2160)**

- Storage project
- Maximum storage = 23.5 KSFD
- Minimum storage = 1.7 KSFD
- Maximum outflow = 1,990 CFS
- Minimum outflow = 100 CFS
- Tailwater elevation = 53.0 feet
- Water conversion factor = 32.0 watt/CFS

- Storage vs. Forebay Elevation for the White River Dam (Ref. 74, Table 157)

3.3.82 **Lagrande (2188)**

- ROR project
- 85% FGFC = 1,888.7 CFS
- Minimum flow = 300.0 CFS

- Generation vs. Outflow Through the Turbine for the Lagrande Dam (Ref. 74, Table 158)

3.3.83 **Alder (2190)**

- Storage project
- Maximum storage = 81.4 KSFD
- Minimum storage = 0.0
- Maximum outflow = 2,610.0 CFS
- Minimum outflow = 300.0 CFS
- Tailwater elevation = 935.0 feet

- Storage vs. Forebay Elevation for the Alder Dam (Ref. 74, Table 159)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Alder Dam (Ref. 74, Table 160)

3.3.84 **Cushman II (2206)**

- ROR project
- 85% FGFC = 2,269.5 CFS
- Minimum outflow = 0.0

- Generation vs. Outflow Through the Turbine for the Cushman II Dam (Ref. 74, Table 161)

3.3.85 **Cushman I (2208)**

- Storage project
- Maximum storage = 187.6 KSFD
- Minimum storage = 0.0
- Maximum outflow = 2,448.0 CFS
- Minimum outflow = 0.0

- Tailwater elevation = 480.0

- Storage vs. Forebay Elevation for the Cushman I Dam (Ref. 74, Table 162)

- Effective Head vs. Water Conversion Factor (Watt/CFS) for the Cushman I Dam (Ref. 74, Table 163)

3.4 The nine reservoir system

- The nine reservoir hydro system is composed of nine projects connected like a tree which is the general case of reservoir topology and it adequately specifies a real system Fig. 6.

CHAPTER IV

Maximum Energy Capability of Hydro-Power System Linear Water Conversion Factor and Constant Tailwater Elevation

In this chapter functional analysis and the minimum norm formulation have been used to maximize the total energy capability for the long-term problem of hydro power systems. The algorithm considers the general configuration of reservoirs and run-of-river topology that one may encounter. The tree connection is the one that can adapt to any arbitrary topological configuration. Calculations have been made for the critical period (CP) which represents a historical stream record during which exogenous inflows to the system is a minimum. The expected consequence of the minimum inflows is the firm total energy capability of the system assuming the system optimum operation has been found. The maximum generation that the hydro-power system can produce during the CP, while optimally drafting the available reservoir storage from full to empty is called the firm energy load carrying capability of the system (FELCC).

The water conversion factor (WCF) is considered as a linear function of the storage. This assumption is much more accurate in representing the true case over other techniques [7,9,50,61] which consider the WCF as a constant. Also, it generalizes the objective function and verifies the well known fact that different heads (storages) mean different generation capability. By WCF we mean the average amount of watts produced due to an outflow of one cubic foot per second.

The time period used for the two systems under consideration is half a month (from 14 to 16 days); and the expectation of the delay in water reaching a plant from upstream plants ranges from zero to about 24 hours; therefore, there will be no significant delay in water reaching a reservoir from its immediate upstream neighbor.

Due to the stochastic nature of exogenous inflows to the system and the planning objective, a stochastic type of optimization technique is used. Many of the well known techniques [7,9,49,50,61,63], cannot accommodate the stochastic nature of the problem.

4.1 Background

Various optimization techniques have been proposed in the past to solve the problem of optimal scheduling of hydro-power systems. The two prominent approaches that have been used in solving the problem are nonlinear programming and dynamic programming.

Nonlinear programming approaches [7,9,26,47,49,50,54,61,63] are developed usually for models with nonseparable benefits. Gagnon et al. 9,50,61] and Hicks et al. [7] worked with the Bonneville Power Authority, Hanscon et al. [63] worked with Hydro -Quebec, Divi et al. [64] worked with ALCAN (Aluminum of Canada) system and Rosethal [49] with Tennessee Valley Authority (TVA). All, but one of these applications required the model to handle a general network topology (any arbitrary topological configuration) for the reservoir system. The exception to this requirement was ALCAN, whose reservoirs exhibits only series arrangements.

The models grouped together above can all be described as having the form

$$\text{maximize} \quad f(y) \tag{4.1}$$

$$\text{subject to} \quad Ay=b \tag{4.2}$$

$$\text{and} \quad \underline{y} \leq y \leq \overline{y} \tag{4.3}$$

where y is a vector of release and storage decision variables. The linear system of equations (4.2) is a set of flow conservation constraints with the vector b a known set of exogenous inflows. The inequality constraints (4.3) place bounds on the flows to serve flood control, navigational, fishing, recreational, physical constraints and other purposes. The great majority of the effort in construction of models of the form (4.1), (4.2), (4.3) is invested in the formulation of the objective function F(y). This step requires, first, the selection of an economic measure of the benefit derived from the system; second, an approximate mathematical representation of the selected measure; and third (usually), an approximation to this is made to make it computationally manageable.

The selected measure in TVA and Hydro-Quebec studies [49,63] was the savings of thermal fuels that result from hydroelectric generation. That is, the economic value of a watt of hydroelectricity is the cost that would have been incurred had the watt been generated from a thermal or nuclear plant instead. In the ALCAN study [64], the measure of benefit was the potential energy in the system at the end of the planning period. In the Bonneville study [7,9,50,61], the measure was a weighted sum of (i) the proportion of power load met with hydro; (ii) the uniformity of load deficits, and (iii) the violation in certain "soft" constraints. The energy production as a function of volume of outflow is taken as a constant [7,9,50,61] or as piece-wise polynomials, with two pieces; one piece represents energy production from outflows that are directed entirely to the turbines, the other piece is needed to account for spills, i.e. situations in which outflows exceed turbine capacities. The ALCAN research team [64] used cubic splines to smooth out the breaks between the two pieces.

The solution approaches in all the nonlinear programming models involve the elimination of some of the variables by means of the equality constraints (4.2). A search direction in terms of the remaining variables (or a subset of the remaining variables) is then computed and used for locating an improved point.

The nonlinear programming algorithms that have been applied to the multireservoir model are fairly efficient in comparison with the dynamic programming approaches. Nevertheless, it does not appear likely that the nonlinear programming methods are adaptable for handling stochastic inflows, because the computational effort already expended in solving the deterministic problems is quite large using the nonlinear programming technique.

Dynamic programming approaches [14,15,16,18,21,51,52,62] to the multireservoir models have as much difficulty as the nonlinear programming approaches in obtaining the stochastic inflows solution. Linear and nonlinear dynamic programming solution methods were used with relatively small systems, or after dividing the given system into small parts, or after applying different methods of aggregations.

The aggregation of a multireservoir system to a single equivalent reservoir and the use of aggregation/decomposition methods

[14,15,16,17,18,25] assured a satisfactory solution for a system where reservoir and inflow characteristics are sufficiently similar to justify aggregation into a single reservoir and hydro-plant model.

Other methods [21] have oversimplified the problem by converting the highly nonlinear problem into a linear one and solving it using linear programming techniques. Others [26] have used quadratic programming techniques.

Christensen and Soliman [23,24,31,32,33,34,35,38,39,41] have used functional analysis and the minimum norm formulation to solve the multireservoir hydro-power problem for relatively small systems (six reservoirs maximum). They solved the problem considering the WCF as a constant multiplied by the net head with purely linear type constraints and applied it to two reservoirs in series [24], then to four reservoirs on three independent system flow rivers [31], and a four reservoir system where each two reservoirs are in series on two independent stream flow rivers [32]. Then they solved the problem considering the WCF as a linear function of the storage with linear type constraints while the objective function is the sum of hydroelectric generation plus the expected future return from the water left in storage at the end of the planning period. They applied the algorithm to a four-reservoir system where each two reservoirs are on an independent river [23] and to a six reservoir system where each two reservoirs are in series on three independent rivers [38]. They also considered the WCF as a quadratic function of the average storage with linear type equality and inequality constraints and applied it to two reservoirs in series [33], to four reservoirs each two in series on two independent rivers [34], and to a six reservoir system where each two lie on an independent river. They then solved the problem when the objective is to obtain a monthly generation equal to a certain percentage of the total hydro-generation of the whole year. The WCF is taken as a quadratic function of the storage at the beginning of the month (the period); and the algorithm is applied to a four-reservoir system each two situated on an independent river [41]. Then they considered WCF as a quadratic function of the average storage [39] and applied the technique to a six-reservoir system where each two reservoirs are located on an independent river.

4.2 Multiobjective Water Management

The main objective of water management here is to answer the question: how much water should be stored in and released from each reservoir of the given system in each period of a given horizon so that system power output is optimum. In spite of this limited scope of decisions, a formidable degree of mathematical complexity can arise.

With the models under consideration, the consequences of a chosen set of values for the decision variables are measured with a scalar-valued objective function. The crux of choosing a specific objective function is to sufficiently aid the planning and operation of water resource systems. There are four important characteristics of reservoir operation problems that are of great importance. The ability or inability to handle these four characteristics is usually a basis for judging different models and algorithms. The characteristics are:

1. the existence of **multiple reservoirs** in the system under study,
2. the need for integrated system management over **multiple time**

periods,

3. the nonseparability in the measurement of the system benefits, and
4. the stochastic nature of exogenous inflows.

It is worthwhile to mention that the specification of multiple reservoirs should perhaps be replaced by mutliple valleys with multiple reservoirs per valley. The point here is that one would desire a model which can handle a complete generality in the topology of the reservoir systems. Some of the multireservoir models in the literature are in fact limited to a single valley (reservoirs in series) or to a single reservoir per valley (reservoirs in parallel) [60].

Of great importance in assessing the strength of the models considered is whether or not the single objective function is allowed to be rather general in form. The specific test of generality usually used is whether or not the single objective function can be nonseparable. A twice differentiable function f(y) is nonseparable (general in form) if for some value of y the Hessian matrix $[\alpha^2 f(y)/\alpha y_i \alpha y_j]$ has some nonzero off-diagonal terms [60]. The importance of nonseparability is that it accounts for the economic interactions of the effects that different decision variables have on the objective function. In the literature of microeconomics and utility theory, separability is called additivity.

4.3 Long-Term Optimal Operation of Hydro-Power Systems

The optimal operation of hydro-power systems can be divided into several subproblems which are more computationally manageable and each subproblem provides the answer to a different aspect of the whole problem. The different subproblems that can be distinguished are as follows:

4.3.1 Long-Term Subproblem

i) Long-term production planning where the hydro-resources utilization is optimized over time periods of 2 to 3 years.

ii) Seasonal production planning; the optimization period here is 12-15 months.

iii) Monthly and half-monthly optimization planning for a one to four year horizon, the optimization period is a month or half a month or a combination of both.

4.3.2 Medium-term Subproblem

i) Weekly production planning, which provides the mode of usage of the available results from the long-term planning to assess the value of reservoir levels at the end of the horizon. The time period of interest is one week.

ii) Daily production planning which provides the answer to two major decisions; unit commitment and economic dispatch. The time period of interest in this case is one day.

4.3.3 Short-Term Subproblem

Short term economic dispatch problem is to determine the loading of all generating sources, active and reactive that are in service at regular time intervals. This is essentially a static optimization problem, requiring recalculation at 10 minute to half hour intervals.

4.4 Hydro Generation

The electric power generated in a hydro plant is a function of discharge through the turbine and the head. The head is the difference between the water level in the reservoir and the level of tail water. The water level is a function of the amount of water in the reservoir. In practice such a relation is always defined for every reservoir. The level of the tail water depends on both the discharge from the reservoir and the level of the next reservoir. The maximum discharge through the plant is defined by the maximum power production of the plant, and the excessive flow beyond this discharge is called forced spill.

The hydro plants on the same river are hydrologically coupled, and they must be operated according to the regulations along the river. These requirements define the acceptable range of the flow through the river which consequently defines the amount of energy production from each plant along the river. The hydro plants with short or long-term reservoirs can take best advantage of the available energy by saving this energy from the low electric demand periods to the high electric demand periods; and from the high at-site inflows periods to the low at-site inflows periods. The hydro plants with small or no reservoir must use this energy as it is available. The utilization of available energy in a river is optimized with respect to all the plants in the river. If there are different owners along the river, such an overall optimization is complex and sometimes impossible.

The long-term and most of the short-term reservoirs break the hydrologic coupling along the river due to their storage capability. These reservoirs decouple, to a certain extent, the upstream plants from the downstream ones. In some publications [64] they are treated as completely separate and independent from the upstream plants; and the river in such cases is divided into several independent segments. Then they solve small problems rather than solving a large one.

4.5 Hydro-Plant Modeling for Long-Term Operation

Hydro power plants are classified into pumped storage plants and conventional hydro plants. The conventional hydroplants are classified into run-of-river plants and storage plants.

4.5.1 Run-of-River Plants

The run-of-river plants have little storage capacity, and use water as it becomes available, water not utilized is spilled. The MWh generated from a run-of-river plant is equal to a constant times the discharge through the turbines

$$G_{k,i} = C_k U_{k,i} \quad \text{MWh} \qquad (4.4)$$

where C_k is a constant measured in MWh/MCF (MCF=10^6ft^3) and referred to as the water conversion factor (WCF); $U_{k,i}$ is the discharge through the turbine for plant k during a period i in MCF; i is an index used for the period number; this period is considered here as half a month (from 14 to 16 days).

4.5.2 Storage Plants

Storage plants are associated with reservoirs with significant storage capacity. In periods with low power requirements water can be stored and then released when the demand is high.

Modeling of storage plants, for a long-term study, depends on the water head variation. For hydro plants in which the water head variation is small, the MWh generated from the plants can be considered as a constant times the discharge, as given in equation (4.4), and this constant is equal to the average number of MWh generated during a period i by an outflow of one MCF. But, for power systems in which the water heads vary by a considerable amount, this assumption is not valid, and the WCF, MWh/MCF, varies with the head, which itself is a function of the storage. In this chapter, the MWh generated can be written as

$$G_{k,i}=E\{[\alpha_k+1/2\beta_k(x_{k,i-1}+x_{k,i})]U_{k,i}\}\ \text{MWh} \qquad (4.5)$$

where α_k and β_k are constants - these can be obtained by least-squares curve fitting to typical plant data available; and $x_{k,i}$ is the storage of plant k at the end of period i in MCF, the symbol E stands for the expected value.

Equation (4.5) is a function of the discharge through the turbines and the average storage between two successive periods i-1 and i, to avoid underestimation for rising water levels and overestimation for falling water levels in the MWh generated.

4.5.3 Pumped Storage Plants

A pumped storage plant is associated with upper and lower reservoirs. During light load periods water is pumped from the lower to the upper reservoir using the available energy from other sources as surplus energy. During peak load the water stored in the upper reservoir is released to generate power that saves fuel costs of the thermal plants. The pumped storage plant is operated until the added pumping cost exceeds the saving in thermal costs.

4.6 Mathematical Model and Optimization Problem*

The mathematical model is based on the following types of relationships and data:

i) basic relationships between the system variables,

ii) water conservation relationships,

iii) relationships which specify the generated power for given flows and heads at each plant in the system,

iv) initial water levels in all system reservoirs,

v) side flows into the system for all time periods,

vi) constraints on generation, flows, contents, and draft. (Some of these constraints are **hard** and others are **soft**, and this is taken into account by the solution method.

The time period used in the modeling is half a month, therefore short range hydraulic and electrotechnic effects are not taken into consideration.

The hydraulic system model is based primarily on tables derived from field measurements and on water balance equations. The latter are simple relationships between contents, total discharge and inflows. The tables consist of the following for each project (Chapter III):

i) Tailwater elevation as a function of total discharge.

ii) Forebay elevation as a function of reservoir contents.

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* A version of this study has been presented as a study report #1 to the B.P.A. Company [Ref. No. 11], and a version of this study has been submitted to Can.Eng.J., June 1987 [Ref. No. 42].

iii) Peak power as a function of head.

iv) Water-to-energy conversion factor as a function of the effective head and turbine discharge (total discharge minus spill), and

v) Seasonal constraints on minimum and maximum storage, outflow, and forebay elevation and maximum draft rate (refill and drawdown).

The constraints reflect physical limits, bank erosion considerations, coordination agreements among various ownerships, system safety and multipurpose requirements such as irrigation, navigation, fishing, flood control, water quality, and recreation.

The expected evaporation and percolation losses are accounted for by deducting them from the forecasted at-site inflows.

4.7 System Under Study

The system under study consists of n reservoirs on the different branches of a river in an arbitrary topological configuration which represents a general case. At each reservoir there is a power plant connected in cascade through power-lines to the neighboring system for energy exchange, Figs. 6 and 7.

To streamline the presentation, it will be assumed that all plants have a variable forebay elevation (i.e. are adjacent to a reservoir), that there is an equal number of reservoirs and plants, that all plants have a storage and that all reservoirs are in the same valley.

The objective of this study is to determine how much water should be stored in, and released from, each reservoir of the given system in each period of a given horizon, so that the system output is optimum.

4.8 Optimization Objective and Constraints

Given a hydroelectric system in an arbitrary topological configuration, the problem is to find the optimal release as a function of time over the given horizon, subject to the following conditions:

i) The total generation from that system over the optimization horizon is a maximum.

ii) The WCF for a reservoir plant is considered as a linear function of the storage, i.e.

$$WCF_{k,i}=\alpha_k+1/2\beta_k\cdot(x_{k,i}+x_{k,i-1}) \tag{4.6}$$

iii) The WCF for run-of-river (ROR) plants is a constant.

iv) To satisfy the constraints that reflect physical limits, bank erosion, coordination agreement among various ownerships and multipurpose requirements such as irrigation, navigation, fishing, flood control, water quality, recreation, and other purposes if any, the plant variables must satisfy the following inequality constraints:

a) upper and lower bounds on reservoir contents,

$$\underline{x}_{k,i}\leq x_{k,i}\leq\overline{x}_{k,i} \tag{4.7}$$

where $\underline{x}_{k,i}$ and $\overline{x}_{k,i}$ are defined to satisfy physical limits, coordination agreements, and multipurpose requirements on reservoir contents.

b) upper and lower bounds on reservoirs and ROR plants outflow,

$$\underline{U}_{k,i}\leq U_{k,i}\leq\overline{U}_{k,i} \tag{4.8}$$

where $\underline{U}_{k,i}$ and $\overline{U}_{k,i}$ are defined to satisfy the system safety, the multipurpose stream use requirements, and coordination agreement on total release.

4.9 Problem Formulation

i) Each station is reduced to a single equivalent input/output curve to reduce the number of variables in the optimization process.

ii) The water conservation equation for each reservoir may be adequately described by the continuity-type equation,

$$x_{k,i}=x_{k,i-1}+INF_{k,i}+\sum_{r\varepsilon R_k} U_{r,i}-U_{k,i}+\sum_{r\varepsilon R_k} S_{r,i}-S_{k,i} \tag{4.9}$$

where R_k is the set of plants immediately upstream of plant k,

$INF_{k,i}$ is the expected natural inflow for plant k during a period i; the expected natural inflows are considered statistically indpendent random variables, and

$S_{k,i}$ is the spillage of plant k during a period i.

The spillage usually causes a negative generation since it raises the tailwater elevation which eventually decreases the effective net head. The net effect is dependent on the design of the hydro-plant. In this chapter, the negative generation of the spillage will be neglected.

iii) The storage plants dictate how the immediate downstream ROR plants operate since in the ROR plants, the total release is equal to the total at-site inflows plus the total release from the upstream plants; and it has no significant storage.

iv) Irrigation, evaporation, and percolation losses are accounted for by deducting them from the forecasted side inflow. In some cases the output of these deductions are negative numbers; in such cases the planner has to wait for the release from the upstream plants or use the available storage to satisfy the minimum flow requirements.

v) In mathematical terms, the optimization objective is to find $U_{k,i}$ that maximizes

$$J = \sum_{i=1}^{m} \sum_{k=1}^{n} G_{k,i}(U_{k,i}, x_{k,i}, x_{k,i-1}) \tag{4.10}$$

subject to satisfying the equality constraints given by equation (4.9), and the inequality constraints given by equations (4.7) and (4.8), using the linear approximation of the WCF, equation (4.6). Symbols m and n stands for the total number of periods in the given horizon and the total number of plants, i.e. the storage plants plus the ROR ones.

4.10 Minimum Norm Formulation*

Substituting from equation (4.5) into equation (4.10) for $G_{k,i}$ we get:

$$J = \sum_{i=1}^{m} \sum_{k=1}^{n} E[\alpha_k . U_{k,i} + 1/2\beta_k (x_{k,i} + x_{k,i-1}) . U_{k,i}] \tag{4.11}$$

Then, substituting from Eq. (4.9) into Eq. (4.11) for $x_{k,i}$ we get:

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* A version of this section has been presented in the North American power Symposium [Ref. No. 4].

$$J=E\left(\sum_{i=1}^{m}\sum_{k=1}^{n}\left[b_{k,i}U_{k,i}+1/2\beta_k U_{k,i}\cdot\left(2x_{k,i-1}+\sum_{r\varepsilon R_k}U_{r,i}-U_{k,i}\right)\right]\right]\tag{4.12}$$

where

$$b_{k,i}=\alpha_k+1/2\beta_k\left(INF_{k,i}+\sum_{r\varepsilon R_k}S_{r,i}-S_{k,i}\right)\tag{4.13}$$

The augmented cost function, $\hat{J}$, is obtained by adjoining to the cost fuction (4.12) the equality constraints (4.9) via Lagrange multipliers and the inequality constraints (4.7) and (4.8) via Kuhn-Tucker multipliers; one thus obtains:

$$\begin{aligned}\hat{J}=E\Big(&\sum_{i=1}^{m}\sum_{k=1}^{n}\Big[b_{k,i}\cdot U_{k,i}+\beta_k\cdot U_{k,i}\cdot x_{k,i-1}+1/2.\beta_k\cdot U_{ki}\cdot\\ &\Big(\sum_{r\varepsilon R_k}U_{r,i}-U_{k,i}\Big)+\lambda_{k,i}\cdot\Big(-x_{k,i}+x_{k,i-1}+INF_{k,i}+\sum_{r\varepsilon R_k}U_{r,i}-U_{k,i}\\ &+\sum_{r\varepsilon R_k}S_{r,i}-S_{k,i}\Big)-e_{k,i}^{1}\left(\underline{U}_{k,i}-U_{k,i}\right)-e_{k,i}^{2}\left(U_{k,i}-\overline{U}_{k,i}\right)\\ &+e_{k,i}^{3}\left(\underline{x}_{k,i}-x_{k,i}\right)-e_{k,i}^{4}\left(x_{k,i}-\overline{x}_{k,i}\right)\Big]\Big]\end{aligned}\tag{4.14}$$

In the above equation $\lambda_{k,i}$ is a Langrange multiplier and will be determined in such a way that the corresponding equality constraints must be satisfied. $e_{k,i}^{1}$, $e_{k,i}^{2}$, $e_{k,i}^{3}$, and $e_{k,i}^{4}$ are Kuhn-Tucker multipliers. They are equal to zero if the constraints are not violated and greater than zero if the constraints are violated.

Equation (4.14) can be written in the following vector form

$$\begin{aligned}\hat{J}=E[\sum_{I=1}^{m} & [b^T(I).U(I)+U^T(I).\beta.x(I-1)+1/2.U^T(I). \\ & \beta.M.U(I)-\lambda^T(I).x(I)+\lambda^T(I).x(I-1)+\lambda^T(I). \\ & INF(I)+\lambda^T(I).M.U(I)+\lambda^T(I).M.S(I)+\theta_1{}^T(I).U(I) \\ & +\theta_2{}^T(I).x(I)]]\end{aligned} \tag{4.15}$$

In the above equation

$$b(I)=col.(b_{1,i},b_{2,i},\ldots,b_{k,i},\ldots,b_{n,i}) \tag{4.16}$$

$$U(I)=col.(U_{1,i},U_{2,i},\ldots,U_{k,i},\ldots,U_{n,i}) \tag{4.17}$$

$$\beta=diag.(\beta_1,\beta_2,\ldots,\beta_k,\ldots,\beta_n) \tag{4.18}$$

$$x(I)=col.(x_{1,i},x_{2,i},\ldots,x_{k,i},\ldots,x_{n,i}) \tag{4.19}$$

$$\lambda(I)=col.(\lambda_{1,i},\lambda_{2,i},\ldots,\lambda_{k,i},\ldots,\lambda_{n,i}) \tag{4.20}$$

$$INF(I)=col.(INF_{1,i},INF_{2,i},\ldots,INF_{k,i},\ldots,INF_{n,i}) \tag{4.21}$$

$$\theta_1(I)=col.(\theta_{1,i}{}^1,\theta_{2,i}{}^1,\ldots,\theta_{k,i}{}^1,\ldots,\theta_{n,i}{}^1) \tag{4.22}$$

$$\theta_2(I)=col.(\theta_{1,i}{}^2,\theta_{2,i}{}^2,\ldots,\theta_{k,i}{}^2,\ldots,\theta_{n,i}{}^2) \tag{4.23}$$

and

$$\theta_{k,i}^{1} = e_{k,i}^{1} - e_{k,i}^{2} \tag{4.24}$$

$$\theta_{k,i}^{2} = e_{k,i}^{3} - e_{k,i}^{4} \tag{4.25}$$

M is an nxn matrix where the diagonal elements are equal to -1 and the other elements vary between 1 and zero depending on the topological arrangement of a given reservoir set.

Example:

the M matrix for the topological arrangement give in Fig. 10 is

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix} \tag{4.26}$$

Constant terms are dropped from Eq. (4.15).

Employing the discrete version of integration by parts so that

$$\sum_{I=1}^{m} X(I) = X(1)+X(2)+\ldots+X(m)$$

$$= X(0)+X(1)+\ldots+X(m)-X(0)$$

$$= \sum_{I=1}^{m} X(I-1)+X(m)-X(0) \tag{4.27}$$

equation (4.15) can be written as

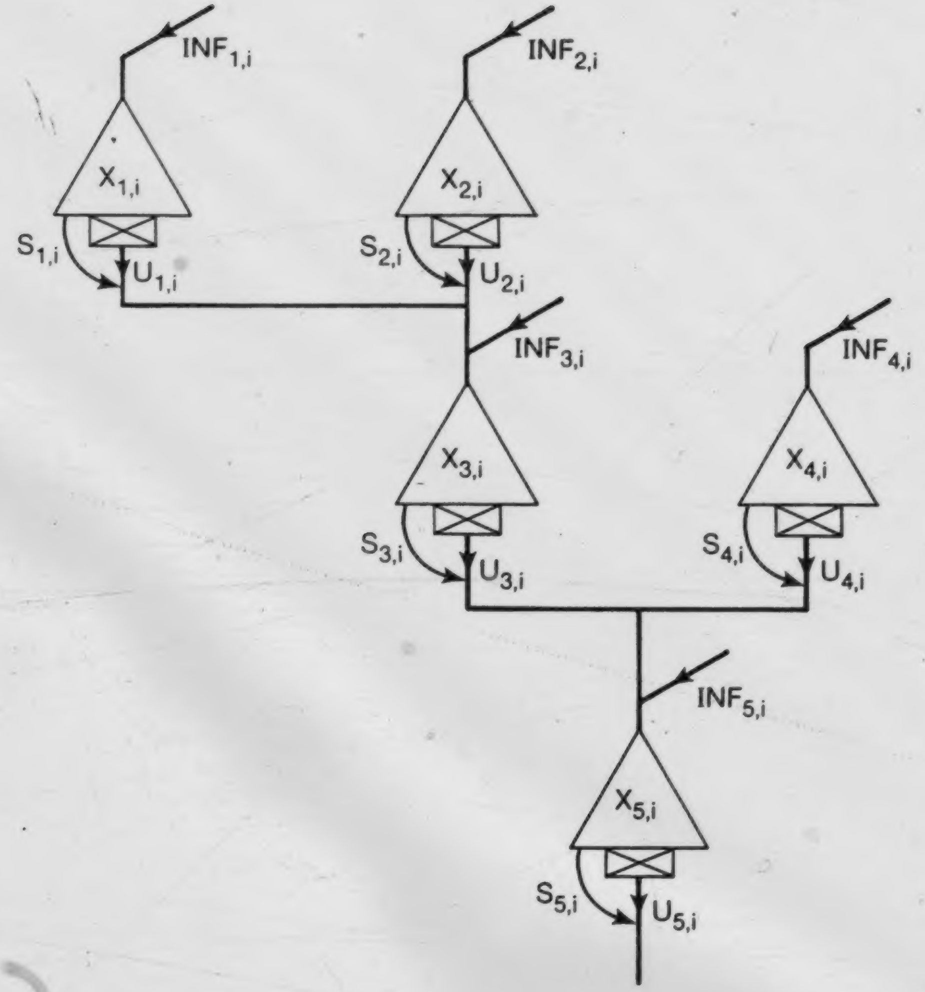
$INF_{1,i}$
$INF_{2,i}$
$X_{1,i}$
$X_{2,i}$
$S_{1,i}$
$U_{1,i}$
$S_{2,i}$
$U_{2,i}$
$INF_{3,i}$
$INF_{4,i}$
$X_{3,i}$
$X_{4,i}$
$S_{3,i}$
$U_{3,i}$
$S_{4,i}$
$U_{4,i}$
$INF_{5,i}$
$X_{5,i}$
$S_{5,i}$
$U_{5,i}$

Fig. 10 Schematic Diagram of a General Configuration of Reservoir Topology for 5-Reservoir System.

$$\hat{J}=E[\lambda^T(0).X(0)-\lambda^T(m).X(m)+\theta_2{}^T(m).X(m)$$

$$-\theta_2{}^T(0).X(0)+\sum_{I=1}^{m}[b^T(I).U(I)=U^T(I).\beta.$$

$$X(I-1)+1/2.U^T(I).\beta.M.U(I)-\lambda^T(I-1).X(I-1)$$

$$+\lambda^T(I).X(I-1)+\lambda^T(I).INF(I)+\lambda^T(I).M.U(I)$$

$$+\lambda^T(I).M.S(I)+\theta_1{}^T(I).U(I)+\theta_2{}^T(I-1).X(I-1)]] \qquad (4.28)$$

Defining the following vectors

$$W^T(I)=[X^T(I-1) \quad U^T(I)] \qquad (4.29)$$

$$R^T(I)=[\lambda(I)-\lambda(I-1)+\theta_2(I-1) \quad b(I)+M^T.\lambda(I)+\theta_1(I)] \qquad (4.30)$$

and

$$L(I)=\begin{bmatrix} 0 & 1/2.\beta \\ 1/2.\beta & 1/4.\beta.M+1/4.M^T.\beta^T \end{bmatrix} \qquad (4.31)$$

one obtains the augmented cost function

$$\hat{J}=E[\theta_2{}^T(m).X(m)-\lambda^T(m).X(m)$$

$$+\sum_{I=1}^{m}[W^T(I).L(I).W(I)+R^T(I).W(I)]] \qquad (4.32)$$

constant parts were dropped from equation (4.32).

Equation (4.32) is composed of a boundary part and a discrete integral part. The two parts are independent of each other. So, Eq. (4.32) can be written as

$$\hat{J}=\hat{J}_1+\hat{J}_2 \tag{4.33}$$

where

$$\hat{J}_1=E[\theta_2{}^T(m).X(m)-\lambda^T(m).X(m)] \tag{4.34}$$

$$\hat{J}_2=E[\sum_{I=1}^{m} (W^T(I).L(I).W(I)+R^T(I).W(I))] \tag{4.35}$$

If we define the vector V(I) such that

$$V(I)=L^{-1}(I).R(I) \tag{4.36}$$

then, the augmented cost function, $\hat{J}_2$, can be written as

$$\hat{J}_2=E[\sum_{I=1}^{m} [(W(I)+1/2.V(I))^T.L(I).(W(I)+1/2.V(I)) - 1/4.V(I)^T.L(I).V(I)]] \tag{4.37}$$

V(I) is independent of the variable vector W(I). Dropping the constant terms from Eq. (4.37) we get

$$\hat{J}_2=E[\sum_{I=1}^{m} [(W(I)+1/2.V(I))^T.L(I).(W(I)+1/2.V(I))]] \tag{4.38}$$

Equation (4.38) defines a norm in Hilbert space, hence we can write Eq. (4.38) as

$$\hat{J}_2 = ||W(I)+1/2V(I)||_{L(I)} \qquad (4.39)$$

4.11 The Optimal Solution

To maximize $\hat{J}$ in Eq. (4.33) we will maximize each term separately.

$$\underset{[X(m),W(I)]}{\text{Max.}}\hat{J} = \underset{[X(m)]}{\text{Max.}}\hat{J}_1 + \underset{[W(I)]}{\text{Max.}}\hat{J}_2$$

The maximum of $\hat{J}_1$ is achieved when

$$\theta_2(m)-\lambda(m) = \underline{0} \qquad (4.40)$$

Eq. (4.40) gives the value of Lagrange multiplier as a function of the Kuhn-Tucker multipliers at the last period.

The maximization of Eq. (4.39) is mathematically equivalent to minimizing the norm of the same equation.* The minimum of Eq. (4.39) is achieved when the norm of this equation is equal to zero; therefore,

$$E[W(I)+1/2.V(I)] = \underline{0} \qquad (4.41)$$

Substituting from Eq. (4.36) into (4.41), we obtain

$$E[R(I)+2.L(I).W(I)] = \underline{0} \qquad (4.42)$$

The minimum of the norm is equal to a vector of zeroes

The maximum of equation (4.39) assured when $E\left\{W(I)+1/2\ V(I)\right\} = \underline{0}$

Eq. (4.42) is the condition of optimality. Writing this equation explicitly and adding the equality constraints, we obtain

$$E[\lambda(I)-\lambda(I-1)+\theta_2(I-1)-\beta.U(I)]=\underline{0} \qquad (4.43)$$

$$E[b(I)+M^T.\lambda(I)+\theta_1(I)+\beta.X(I-1)+1/2.\beta.M.U(I)$$

$$+M^T.\beta^T.U(I)]=\underline{0} \qquad (4.44)$$

$$E[-X(I)+X(I-1)+INF(I)+M.U(I)+M.S(I)]=\underline{0} \qquad (4.45)$$

$$E[-b(I)+\alpha+1/2.\beta.INF(I)+1/2.\beta.M.S(I)]=\underline{0} \qquad (4.46)$$

$$-\theta_1(I)+e_1(I)-e_2(I)=\underline{0} \qquad (4.47)$$

$$-\theta_2(I)+e_3(I)-e_4(I)=\underline{0} \qquad (4.48)$$

where

$$\alpha=col.(\alpha_1,\alpha_2,\ldots,\alpha_k,\ldots,\alpha_n) \qquad (4.49)$$

$$e_1(I)=col.(e_{1,i}^{1},e_{2,i}^{1},\ldots,e_{k,i}^{1},\ldots,e_{n,i}^{1}) \qquad (4.50)$$

$$e_2(I)=col.(e_{1,i}^{2},e_{2,i}^{2},\ldots,e_{k,i}^{2},\ldots,e_{n,i}^{2}) \qquad (4.51)$$

$$e_3(I)=col.(e_{1,i}^{3},e_{2,i}^{3},\ldots,e_{k,i}^{3},\ldots,e_{n,i}^{3}) \qquad (4.52)$$

$$e_4(I)=\text{col.}(e_{1,i}^{\;4},e_{2,i}^{\;4},\ldots,e_{k,i}^{\;4},\ldots,e_{n,i}^{\;4}) \qquad (4.53)$$

We also have the following limits on Kuhn-Tucker values

$$e_{k,i}^{\;1}=\begin{cases} 0.0 \\ >0.0 \end{cases} \quad \text{if} \quad \begin{cases} U_{k,i}\geq \underline{U}_{k,i} \\ U_{k,i}<\underline{U}_{k,i} \end{cases} \qquad (4.54)$$

$$e_{k,i}^{\;2}=\begin{cases} 0.0 \\ >0.0 \end{cases} \quad \text{if} \quad \begin{cases} U_{k,i}\leq \overline{U}_{k,i} \\ U_{k,i}>\overline{U}_{k,i} \end{cases} \qquad (4.55)$$

$$e_{k,i}^{\;3}=\begin{cases} 0.0 \\ >0.0 \end{cases} \quad \text{if} \quad \begin{cases} X_{k,i}\geq \underline{X}_{k,i} \\ X_{k,i}<\underline{X}_{k,i} \end{cases} \qquad (4.56)$$

$$e_{k,i}^{\;4}=\begin{cases} 0.0 \\ >0.0 \end{cases} \quad \text{if} \quad \begin{cases} X_{k,i}\leq \overline{X}_{k,i} \\ X_{k,i}>\overline{X}_{k,i} \end{cases} \qquad (4.57)$$

Equations (4.43) to (4.57) with Eq. (4.40) completely specify the optimal solution.

4.12 Algorithm for Solution

Given a system of n reservoirs, the expected values of natural inflows $INF_{k,i}$ for each plant $k=1,\ldots,n$, at each period $i=1,\ldots,m$, the initial storage $X_k(0)$ (all reservoirs must be full at the beginning of

CP, $X_k(0)=\bar{X}_k(0)$, unless drafting for minimum flow or flood control), tables which specify the relationships between the variables at each plant in the system, and different kinds of constraints.

1) First: assume initial values for $X_{k,i}$, $k=1,\ldots,n$; $i=1,\ldots,m$ then, check the limits on $X_{k,i}$, so that

if $X_{k,i}<\underline{X}_{k,i}$ let $X_{k,i}=\underline{X}_{k,i}$
if $X_{k,i}>\bar{X}_{k,i}$ let $X_{k,i}=\bar{X}_{k,i}$

2) Start with spillage $S_{k,i}$ equal to zero and calculate $U_{k,i}$ using Eq. (4.9), the water conservation equation starting from the end reservoir or ROR plant and then go to the next down-stream plant till the end of the branch taking one branch at a time.

3) Check the limits on $U_{k,i}$

if $U_{k,i}<\underline{U}_{k,i}$ let $U_{k,i}=\underline{U}_{k,i}$ and calculate a new $X_{k,i}$
if $U_{k,i}>\bar{U}_{k,i}$ let $U_{k,i}=\bar{U}_{k,i}$ and calculate a new $X_{k,i}$

4) Check the limits on $X_{k,i}$

if $X_{k,i}<\underline{X}_{k,i}$ a) let $X_{k,i}=\underline{X}_{k,i}$
b) calculate the corresponding value of $U_{k,i}$
c) let $e_{k,i}^{1}>0.0$
if $X_{k,i}>\bar{X}_{k,i}$ a) let $X_{k,i}=\bar{X}_{k,i}$
b) let $U_{k,i}=\bar{U}_{k,i}$
c) calculate the corresponding value of $S_{k,i}$
d) let $e_{k,i}^{2}>0.0$

If $X_{k,i}$ and $U_{k,i}$ satisfy the boundary constraints then the corresponding values of Kuhn-Tucker multipliers will be set equal to zero, Eqs. (4.54) to (4.57).

5) Calculate the values of $\lambda_k(m)$ from Eqs. (4.40) and (4.48).

6) Calculate the values of $\lambda_k(i)$ from Eqs. (4.43) and (4.48) backward using the values of $\lambda_k(m)$ from step (5).

7) Determine the new updated value of $X_{k,i-1}$ using Eqs. (4.44), (4.46) and (4.47). Then adjust any violation on $X_{k,i}$ so that $\underline{X}_{k,i} \leq X_{k,i} \leq \overline{X}_{k,i}$.

8) If the solution **converges** toward a better solution (this can be observed by calculating the objective function), then continue (go to step 2) until no significant changes occur from iteration to iteration. If the solution **diverges** then, first stop the calculation. Second use the results to modify the controlling soft constraints, $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$ which will be discussed in the next section. Then, repeat the calculations.

4.13 Global Maximum

Since the starting point has a considerable influence on the number of iterations leading to a convergent solution which is not guaranteed to be the global maximum, and since the system different starting points can be infinite even for experienced system engineers, it is essential to search for a way to cope with this general problem which has faced all nonlinear optimizers until now.

Techniques that use trial and error with different starting points [5,31,32,33,34,35,37,38,39,40,41,45,46] may be acceptable for a small number of reservoirs and ROR plants, up to 6 projects, but these techniques cannot be used for large systems such as the BPA hydro system.

The proposed technique searches for the optimum solution through the expected or recommended discharge. This is done by using two new values for maximum discharge and maximum storage, $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$, for all reservoirs during all periods. The proposed technique will deal with these two new values, $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$, as the new maximum values of the discharge and reservoir content. These new values will be considered as a soft constraint i.e., the solution algorithm may omit any or both at any period if the recommended values cause any violation of a hard constraint or cause a spill at that period. In this case (when the system violates any or both of the recommended soft constraints, $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$) the system will remain at the given values of maximum discharge, $U_{k,i}$, and maximum reservoir content, $\overline{X}_{k,i}$. The recommended new soft constraint values should be within the acceptable range of the hard constraints, i.e.,

$$\underline{U}_{k,i} \leq \hat{U}_{k,i} \leq \overline{U}_{k,i}$$

and

$$\underline{X}_{k,i} \leq \hat{X}_{k,i} \leq \overline{X}_{ki,i}$$

The values of $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$ are defined by the programmer who supposedly has some idea about the whole system or parts of the system. The programmer in this case will suggest the ranges which he expects contain the global maximum during that horizon; in choosing these ranges he may use any previous studies, any recommendations, and the supplied soft constraits from different ownerships. But, the optimizer should be cautious in ending his recommended range with a one point

Table 1

Water Conversion Factor as a Linear Function of the Storage for the Nine Reservoir System

Plant No.	WCF=$\alpha+\beta x$ MW/MCF	
	α	β
1	9.102	6.712
2	7.410	3.020
3	2.090	4.200
4	3.1367	1.7333
5	0.948	3.010
6	0.285	3.670
7	4.950	7.150
8	5.303	1.300
9	1.7833	7.830

Table 2

Maximum Discharge Capacity of the Power House for Each Project of the Nine Reservoir System

Project No.	Max.Discharge (CFS)
1	165,000.0
2	56,600.0
3	225,400.0
4	216,320.0
5	40,700.0
6	44,600.0
7	998.000.0
8	31,200.0
9	232,000.0

Table 3

Minimum Discharge Requirements for each Project of the Nine Reservoir System

Project No.	Min.Discharge (CFS)
1	3,000.0
2	0.0
3	5,000.0
4	3,000.0
5	100.0
6	500.0
7	*
8	50.0
9	**

* varies during the year, Table 4

** varies during the year, Table 5

Table 4

Minimum Flow Requirement for Project No. 7 of the Nine Reservoir System

Period from	to	Min.Flow (CFS)
Apr. 1	Aug. 31	30,000.0
Sept.1	Dec. 31	50,000.0
Jan. 1	Feb. 28	30,000.0
Mar. 1	Mar. 31	50,000.0

Table 5

Minimum Flow Requirement for Project No. 9 of the Nine Reservoir System

Period from	to	Min.Flow (CFS)
Dec. 1	Feb. 28	12,500.0
Mar. 1	Sept.30	50,000.0

Table 6

Maximum Storage Requirement for Each Project of the Nine Reservoir System

Project No.	Max.Storage (KSFD)
1	6,073.0
2	557.0
3	3,579.6
4	2,510.5
5	705.8
6	296.9
7	2,614.3
8	341.5
9	269.7

Table 7

Minimum Storage Requirement for each Project of the Nine Reservoir System

Project No.	Min.Storage (KSFD)
1	543.8
2	0.0
3	0.0
4	0.0
5	0.0
6	69.0
7	0.0
8	0.0
9	0.0

Table 8

Expected Natural Inflow During One Dry Year (24 Period)
For Each Project of the Nine Reservoir System

Period		Expected Natural Inflow (MCF)								
		Reservoir Number								
Period	#	1	2	3	4	5	6	7	8	9
July 1-15	1	76959	30197	31663	27167	13219	24997	74639	3745	45140
July 16-31	2	82090	32210	33773	28978	14101	26664	79615	3995	48149
Aug. 1-15	3	70333	19881	21893	10916	8300	10826	18379	1558	23333
Aug. 16-31	4	41706	11999	13068	6874	5253	8014	25355	1388	18873
Sept. 1-15	5	32734	11068	19808	8860	3135	8615	15625	892	16787
Sept.16-30	6	16405	5521	9907	7313	2464	5183	14082	715	27254
Oct. 1-15	7	10303	3328	12453	8903	2320	7359	19890	1192	35202
Oct. 16-31	8	10990	3550	13284	9497	2475	7849	21216	1271	37549
Nov. 1-15	9	6376	2955	7281	5533	1295	5736	19711	531	46843
Nov. 16-30	10	6376	2955	7281	5534	1295	5736	19711	531	46843
Dec. 1-15	11	5495	3214	3059	4199	810	4456	16282	570	38119
Dec. 16-31	12	5861	3428	3263	4479	864	4753	17367	608	40661
Jan. 1-15	13	5586	4549	0	3525	498	2850	11479	493	35781
Jan. 16-31	14	5958	4852	0	3760	531	3040	12244	525	38167
Feb. 1-14	15	3580	2020	750	3544	385	2262	12786	411	34320
Feb. 15-28	16	3580	2020	750	3544	385	2262	12786	411	34320
Mar. 1-15	17	4121	1737	3149	3797	508	5272	23592	498	61914
Mar. 16-31	18	4396	1852	3359	4050	542	5624	15165	531	66041
Apr. 1-15	19	2779	1500	6363	3827	540	5574	25460	499	46122
Apr. 16-30	20	4764	2544	10847	6541	1173	10831	50076	1383	67490
May 1-15	21	22412	13738	26131	24186	5029	30917	101766	4741	98375
May 16-31	22	23906	14653	27873	25798	5364	32979	108550	5057	104934
June 1-15	23	71168	33048	37773	43874	11962	38342	104224	57222	119768
June 16-30	24	71168	33048	37773	43874	11962	38342	104224	57222	119768

range (one feasible point), because this implies a pre-estimated solution for that part of the system during that period which may not guarantee a system global maximum.

Using this algorithm helps to assure a conversion in a minimum time. The optimizer may have to modify the values of $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$ after each trial especially those values which have been violated until he reaches what is believed to be the global maximum.

4.14 Nine-Project System

The algorithm is used successfully to optimize the total release of each reservoir during each period (half a month periods) for a nine-reservoir hydro system, Fig. 6, for one dry year (24 periods). The water conversion factor in a given period is taken as a linear function of the average reservoir content during that period. Table 1 includes the values of the reservoir constants α and β, watt hour per cubic feet and watt hour per cubic feet square respectively. The definitions of α and β are given in Eqs. (4.5) and (4.6). The projects' minimum and maximum discharge during each period are given in tables 2 and 3. The projects minimum and maximum content during each period are given in tables 6 and 7. The expected natural inflows during the 24 periods for each reservoir are given in table 8.

The program takes 0.4 seconds of CPU time on the MTS-AMDAHL-5870 computer system for each trial to calculate the optimal operation of the nine reservoir system. I have done 7 trials. After each trial I modified the values of $U_{k,i}$ and $X_{k,i}$.

The results show a significant increase in the total system energy capability over any starting point and completely satisfy the system constraints.

Tables 9 and 10 contain the final rule-curves for the given system. Table 9 includes the reservoir contents during each period of the optimization horizon (24 period). In table 10 total generation and average power during different iterations is given.

Table 9

Reservoir Contents (MCF) During Each Period of the One Year Optimization Horizon

Reservoir Content $X_{k,i}$

Period	RESERVOIR No.								
	1	2	3	4	5	6	7	8	9
1	524710	48125	309280	216910	60981	25652	225880	29506	23302
2	524710	48125	309280	216910	60981	25652	225880	29506	23302
3	524710	48125	309280	216910	60981	24881	225620	29506	23302
4	524710	48125	309280	216910	60981	25652	224850	29506	23302
5	524700	48125	303090	216910	60981	5962	225880	29009	23302
6	524700	48125	270440	216110	54432	5962	196590	28329	23302
7	484880	48125	261360	213110	48739	5962	188540	28126	23302
8	440870	48125	259990	210670	43289	5962	179230	27999	23302
9	392230	48125	257000	204220	35085	5962	141890	27131	23302
10	329640	48125	256920	197710	31400	5962	125340	26251	23302
11	276180	48125	251480	189920	27216	5962	130330	25809	23302
12	222910	48125	246860	182420	24100	5962	135460	25218	23302
13	169500	48125	239040	174000	20495	5962	134890	24311	23302
14	116540	48125	231170	161770	17003	6516	131410	23447	23302
15	61117	48125	220560	132130	12898	9586	153120	22459	23302
16	46984	40882	175080	121680	9180	13802	154910	21476	23302
17	46984	31494	122620	104490	5586	18042	174450	20588	23302
18	46984	25715	66262	48518	2140	25652	142270	19714	23302
19	46984	8919	21566	0	0	25652	84932	4194	23302
20	46984	0	0	0	0	5962	20	0	23302
21	46984	0	0	0	0	8292	0	0	23302
22	46984	0	0	0	0	10777	0	0	23302
23	46984	31641	0	31690	0	15463	0	0	23302
24	46984	48125	0	0	0	25652	0	0	23302

Table 10

Total Generation During Different Iterations of the Optimization Process

	Total Generation	
Iteration No.	Energy MWH	Power MW
1	5.9788E+04	6.8251E+00
2	5.9503E+04	6.7926E+00
3	6.5606E+04	7.4893E+00
4	6.5609E+04	7.4896E+00
5	6.5609E+04	7.4896E+00
6	6.5609E+04	7.4896E+00
7	6.5609E+04	7.4896E+00
8	6.5609E+04	7.4896E+00
9	6.5609E+04	7.4896E+00

4.15 The B.P.A. System

The problem of scheduling the hydro-power of the B.P.A. system (see schematic diagram in Fig. 8, and section 3.3, Chapter III) considering a constant tailwater elevation and a WCF as a linear function of the storage is presented in this chapter. Maximum draft constraints (nonlinear state dependent inequality type of constraints) have not been considered. Only linear type of constraints have been taken into consideration.

The B.P.A. hydro system consists of 37 storage projects and 51 ROR projects, Fig. 8. The storage capacities of the reservoirs vary between 3579.6 KSFD (Arrow), 2614.3 KSFD (Grand Coulee) to 1.4 KSFD (Packwood L.). Also, the power houses capacities vary between 6684.0 MW (Grand Coulee), 2686.8 MW (Chief Josephy), 6.0 MW (Monroe Streket) to zero MW or no power house at all (Priest Lake). The maximum discharge capacity through the turbine also varies between 375,000.0 CFS (The Dalles), 308,500.0 CFS (Bonneville), 14,576.0 (Mayfield) to 275.0 CFS (Packwood).

Because of the nature of some projects, six projects have been grouped so that each two are considered as one project. This will decrease the number of calculations without any effect on the individuality of each of them. The first two projects are Packwood-62, and Packwood L.-63, see section 3.3.3 in Chapter III. Packwood-62 is a ROR project which has:

minimum outflow requirements = 0.0 CFS

maximum outflow through the turbine = 275.0CFS

maximum power capability = 30.0 MW

and generation versus outflow through the turbine are given in table 6, Ref. 74.

Packwood L.-63 is a storage project which has:

minimum outflow requirements = 10.0 CFS

no power house for generation

maximum storage capacity = 1.4 KSFD

Combining the two projects result in the one given in section 3.3.3, Chapter III. As can be observed the combined constraints satisfy both project constraints and it is easy to see from the results for the combined project that the rule curves for each one can easily be found.

The next two projects are Post Falls and Cour D'Alene Lake. Post Falls is a ROR project which has:

a minimum flow requirement of 0.0 CFS

a maximum outflow through the turbine of 4,700.0 CFS

a maximum power capability of 16.0 MW

and a generation versus outflow through the turbine as given in table 98, Ref. 74.

Cour D'Alene Lake is a storage project which has:

minimum flow requirement = 50.0 CFS

maximum outflow constraints =15,000.0 CFS

no power house for generation

and maximum storage capacity = 112.5 KSFD.

Combining the two projects together result in the one given in section 3.3.53, Chapter III.

The third two projects are Col.Falls and Hungry Horse. Col. Falls is a ROR project which has:

minimum flow requirement = 0.0 CFS

no maximum flow constraints

no power house for generation

Hungry Horse is a storage project which has:

minimum flow requirement = 400 CFS

maximum flow constraints as given in Table 118, Ref. 74.

storage vs. forebay elevation as given in table 119, Ref. 74.

tailwater elevation vs. total outflow as given in table 120, Ref. 74.

effective head vs. WCF as given in table 121, Ref. 74.

Combining the two projects results in the one given in section 3.3.64, Chapter III. As we can see, the combined constraints satisfy both the two projects constraints. Also, it is easy to determine the operating strategy of any of them from the results of the combined project.

In the case of Canal Plant and Corra Linn projects, Fig. 9, a special subroutine has been written to suite their special configuration and the required distribution of water budget between them, table 126, Ref. 74.

The computation process is performed for 96 time periods (four years) including 84 critical time periods (periods 5 to 88 inclusive).

The results show: no hard constraint violation at all, very small amount of spill results compared to the B.P.A. rule curves, and most importantly, an increase in the system total energy capability of 4.52%. Table 11 compares the average power generated by the B.P.A. rule curves and the calculated ones. In table 12, a comparison of the energy capability of the whole system during the total horizon (42-month critical period) between the B.P.A. rule curves and the calculated ones is presented.

Table 11

A Comparison of the Average Power Generated Between the B.P.A. Rule Curves Results and the Calculated Ones Each Year For the 42-Month Critical Period

Period		Av. Power Generated for	
from	to	B.P.A.	Calculated
		Rule Curves in GW	
Sept. 1, 1928	June 31, 1929	12,313	13,054
July 1, 1929	June 31, 1930	11,730	12,419
July 1, 1930	June 31, 1931	12,254	12,427
July 1, 1931	Feb. 29, 1932	12,076	12,723

Table 12

A Comparison of the Energy Capability of the Whole Hydro System During the Total Horizon (42-Month Critical Period) Between the B.P.A. Rule Curves Results and the Calculated Ones

Comparison Between	Rule Curves	
	B.P.A.	Calculated
Total energy (GWH)	370,361	387,088
Average power (GW)	12.0844	12.6301
% Increase	4.51622%	

4.16 Conclusions

A presentation has been made to illustrate the solution for the half month operating policy of a multireservoir-tree connected hydro-electric power system with what is believed to be one of the largest hydro-electric nonlinear optimization problems attempted considering the number of variables and constraints.

5

The proposed solution has been done by a one-at-a-time technique. It starts from the end reservoir or ROR plant of each branch and follows one branch at a time. The problem is formulated as a minimum norm problem and solved using functional analysis and the minimum norm formulation techniques. The time period used here is half a month; therefore, short range hydraulic and electrotechnique effects are not taken into consideration.

The tree system is a general case of the reservoir topology which adequately specifies any real system. It is an improvement over the previous methods which deal with independent rivers with several reservoirs in series or in parallel. Also, it is an improvement over the methods which need essentially good initial estimates.

The basic feature of this new procedure is its ability to automatically produce maximum hydro-generation while satisfying the system constraints.

The technique overcomes the influence of starting points and is able to combine methodology and experience to end with the system global ma[illegible]mum.

CHAPTER V

Long-Term Optimal Operation of Hydro-Power Systems

In the previous chapter the long-term optimal operation problem for hydro-power systems with any arbitrary topological configuration (the tree connection fits any arbitrary topological arrangement of hydro-systems) has been solved. All types of constraints are considered except the maximum draft constraints (nonlinear state dependent inequality type constraints). The water conversion factor (WCF) considered in the previous chapter is a linear function of the effective head (effective head equals the forebay elevation minus the tailwater elevation). Also in Chapter IV, tailwater elevation is considered as a constant equal to the average value of the tailwater variations. Moreover, we have assumed that the storage is a linear function of the forebay elevation. Even though the study in Chapter IV has many advantages over previous studies, it still has many inaccuracies caused by: i) considering constant tailwater elevation, ii) considering WCF as a linear function of the storage, iii) considering storage as a linear function of forebay elevation, and iv) considering only the linear types of constraints.

In this chapter the same problem will be solved considering a much more accurate and acceptable representation of the system. The WCF will be taken as a quadratic function of the storage, and in section 5.2 we will see that this assumption is highly advantageous. All types of constraints will be considered including maximum draft constraints (nonlinear inequality type of constraints). Also, in this chapter, tailwater elevation is a variable and represents a nonlinear function of total release (discharge through the turbine plus spillage plus bypass) and is solved using a suitable cubic spline subroutine (Appendix 1). In addition to the previous assumption forebay elevation will be considered as a nonlinear function of the storage and will be solved for by using a suitable cubic spline subroutine.

The resulting problem of scheduling the operation of a hydro-power system is a formidable problem since: (1) its objective function is highly nonlinear, (2) the production energy function of the hydro-plants is a non-separable function of the discharge and the effective head which itself is a function of the storage, (3) there are linear and nonlinear inequality constraints on both the state (storage) and the decision (release) variables, (4) it is a stochastic problem with respect to the river flows, and (5) the availability of limited amounts of hydroelectric energy, in the form of stored water in the system reservoirs, makes the optimal operation problem very complex because it creates a link between an operating decision in a given stage and the future consequences of this decision. The complexity of the operation problem can easily be seen if we know that none of the given references deal simultaneously with all aspects of the problem (multiple reservoirs, multiple periods, stochastic inflows, and nonseparable benefits) described in the previous chapter.

The solution model uses functional analysis and the minimum norm formulation technique (Chapter II), to search for the changes in storage values for each reservoir, during each period, that will increase the generation under the given critical water condition, with a complete satisfaction of the system hard constraints. There are two types of constraints; hard and soft. Constraints which derived from physical properties of the system and those which cannot be violated under any circumstances are called the hard constraints. The remaining are soft constraints and are expressions of desired operating ranges which can be violated to some extent. A totally feasible solution (i.e., one which strictly satisfies all hard and soft constraints) may not exist for all problems posed. The ability of the proposed technique to produce the maximum energy capability while satisfying the system hard constraints is evident and the results show much promise.

5.1 Background

The problem of optimizing the operation of a hydro-power system is a stochastic nonlinear, discrete-time problem. It is a problem of allocating limited resources in a highly constrained environment that reflects: physical limits, bank erosion, coordination agreements among various ownerships, and multipurpose stream flow requirements such as: irrigation, navigation, fishing, water quality, flood control, and recreational activities.

Many techniques for obtaining the optimal operation for hydro-power systems have been developed and yet no completely satisfactory solution has been obtained, since in every publication the problem is over-simplified in order to cope with the complications and dimensionality of the system. Among these different approaches that simplify the topology of the actual systems we can distinguish the following methods:

(1) The decomposition approach

(2) The aggregation approach

(3) The aggregation/decomposition approach

(4) The successive approximations approach

Also, we can classify the methods of solution as follows:

(1) Dynamic programming methods

(2) Linear, quadratic, and nonlinear programming methods

(3) Discrete maximum principle methods

(4) Functional analysis and the minimum norm formulation methods

Furthermore, all these methods of solution can handle the problem as a:

(1) Deterministic problem, or

(2) Stochastic problem

5.1.1 Simplified System Topology

5.1.1.1 The Decomposition Approach

The procedure first builds a composite reservoir for all the downstream reservoirs k+1,...,m, and then represents the whole system by two reservoirs the reservoir k and the composite reservoir. The remaining reservoirs 1,...,k-1 have already been solved for and are considered as a constant supply of power to the system (usually taken as a negative load) and inflow to downstream projects. Finally, the problem of determining the optimal operating policy of a two-reservoir system is easily solved using any technique of solution (usually dynamic programming is used).

The Composite Model [15,16]

To build a composite model first, assign a fixed WCF MWh/ft^3 to each hydro-plant. This factor is taken equal to the average MWh produced in a period k by an outflow of one cubic foot.

Second, the water stored in each reservoir is converted to energy by multiplying it by the sum of the conversion factors of the downstream plants. Then the total potential energy$XP_{k+1,i}$ stored in reservoirs k+1 to n at the end of the period i is given by

$$XP_{k+1,i} = \sum_{j=k+1}^{n} \sum_{\ell=j}^{n} WCF_{\ell} \cdot X_{j,i} \tag{5.1}$$

where WCF_l is the water conversion factor measured by MWh/ft^3 at site ℓ, and $X_{j,i}$ is the water content of reservoir j at the end of period i. Similarly, the inflow potential energy to reservoirs k+1 to m in period i is

$$INFP_{k+1,i} = \sum_{j=k+1} \sum_{\ell=j}^{n} WCF_{\ell} \cdot INF_{j,i} \tag{5.2}$$

where $INF_{j,i}$ is the at-site total inflow to plant j during a period i. The outflow potential energy from reservoir k+1 to m in period i is

$$UP_{k+1,i} = \sum_{j=k+1}^{n} WCF_j \cdot U_{j,i} \tag{5.3}$$

Equations (5.1), (5.2) and (5.3) completely define a composite model for reservoirs k+1 to n.

In practice, the MWh generated from the composite reservoir is not equal to the actual MWh generated from each reservoir. The first reason for this is that the actual water conversion factor, MWh/ft^3 at a plant depends heavily on the net head, which may vary considerably. A second reason is that there may be spillage at some plants. Therefore a generating function, $G_{k+1,i}(XP_{k+1,i}, UP_{k+1,i})$, must be constructed that relates the actual generation to the MWh (outflow and energy content) of the composite reservoir.

The decomposition approach may be justified only for similar reservoirs; otherwise the solution obtained by this method is not a global feedback solution, but it is rather a suboptimal operating policy, because it is not yet possible to obtain global feedback solutions for large-scale systems using this method.

5.1.1.2 The Aggregation Approach

The systems usually used in this approach have p independent rivers [17,18]; the following assumptions are used:

(1) The optimal operating policy of river j is such that spillage will not occur in period i nor will it occur at all the reservoirs on the river; and shortage of water will not occur in period i nor will it occur in all the reservoirs on the river.

(2) The amount of electric energy generated by any power plant in period i is a constant times the discharge.

Following these hypotheses, a composite model of each river is then built. The resulting composite model of river j has the following characteristics:

$$INF_{j,i} = \sum_{k=1}^{n} \sum_{\ell=k}^{n} WCF_{\ell,j} \cdot INF_{\ell,j,i} \qquad (5.4)$$

where $INF_{j,i}$ is the inflow potential energy of composite reservoir j in period i, $INF_{k,j,i}$ is the inflow of water to reservoir k on river j during a period i, and $WCF_{\ell,j}$ is the water conversion factor, MWh/MCF of water at site ℓ of river j.

$$X_{j,i} = \sum_{k=1}^{n} \sum_{\ell=k}^{n} WCF_{\ell,j} . X_{k,j,i} \qquad (5.5)$$

$$\underline{X}_{j,i} \leq X_{j,i} \leq \overline{X}_{j,i} \qquad (5.6)$$

where

$$\underline{X}_{j,i} = \sum_{k=1}^{n} \sum_{\ell=k}^{n} WCF_{\ell,j} . \underline{X}_{\ell,j,i} \qquad (5.7)$$

$$\overline{X}_{j,i} = \sum_{k=1}^{n} \sum_{\ell=k}^{n} WCF_{\ell,j} . \overline{X}_{\ell,j,i} \qquad (5.8)$$

$X_{j,i}$ is the storage potential energy of composite reservoir j at period i,

$\overline{X}_{k,j,i}$ is contents of reservoir k on river j at the end of period i,

$\underline{X}_{j,i}$ is the storage minimum potential energy of composite reservoir j at period i,

$X_{j,i}$ is the storage maximum potential energy of composite reservoir j at period i,

$\underline{X}_{k,j,i}$ is the minimum storage constraint of reservoir k on river j at period i,

$\overline{X}_{k,j,i}$ is the maximum storage constraint of reservoir k on river j at period i

$$U_{j,i} = \sum_{k=1}^{n} WCF_{k,j} \cdot U_{k,j,i} \qquad (5.9)$$

$$\underline{U}_{j,i} \leq U_{j,i} \leq \overline{U}_{j,i} \qquad (5.10)$$

$$\underline{U}_{j,i} = \sum_{k=1}^{n} WCF_{k,j} \cdot \underline{U}_{k,j,i} \qquad (5.11)$$

$$\overline{U}_{j,i} = \sum_{k=1}^{n} WCF_{k,i} \cdot \overline{U}_{k,j,k} \qquad (5.12)$$

where $U_{j,i}$ is the discharge potential energy of composite reservoir j at period i,

$U_{k,j,i}$ is the discharge potential energy of composite reservoir j at period i,

$U_{k,j,i}$ is the discharge from reservoir k on river j during period i,

$\underline{U}_{j,i}$ is the discharge minimum potential energy of composite reservoir j at period i,

$U_{j,i}$ is the discharge maximum potential energy of composite reservoir j at period i,

$\underline{U}_{k,j,i}$ is the minimum discharge of reservoir k on river j at period i,

$\overline{U}_{k,j,i}$ is the maximum discharge of reservoir k on river j at period i,

and

$$S_{j,i} = \sum_{k=1}^{n} WCF_{k,j} \cdot S_{k,j,i} \qquad (5.13)$$

where $S_{j,i}$ is the amount of potential energy spilt from composite reservoir j at period i,

$S_{k,j,i}$ is the spillage from reservoir k on river j during period i.

If we replace every river by its composite model, we obtain a network of p reservoir hydro-plant complexes in parallel.

The main drawback of this method is that the approach avoids answering the basic question of how the individual reservoirs in the system are to be operated in an optimal fashion.

5.1.1.3 The Aggregation Decomposition Approach

The optimization of a system of m reservoirs is broken down here to m subproblems in which one reservoir is optimized knowing the total energy content for the rest of the reservoirs. For each subproblem one of the reservoir hydro-plant models is retained and the remaining m-1 are aggregated into an equivalent reservoir hydro-plant model. The global feedback characteristic of the problem is thus retained and the technique can potentially handle all the uncertainties as well as the local constraints in each hydro chain. Furthermore, the computational requirement of this method grows linearly with the number of reservoirs. More precisely, for each new reservoir added to the system, only one additional demand of two-state variables needs to be solved.

5.1.1.4 The Successive Approximations Approach

The successive approximations approach uses the local feedback policy in which each reservoir is optimized independently assuming an expected operation (state or release) of the rest of the reservoirs. The procedure iterates, using a one-at-a-time optimization technique for each reservoir, until convergence is found. Detailed representations of each hydro chain can be used (random inflow, serial correlation, local constraints, etc.). The major drawback of this approach is that it ignores the dependence of the operating policy of one reservoir on the actual energy content of other reservoirs. The method gives good results only if the actual operation of the remaining reservoirs is very close to the expected values.

5.1.2 Methods of Solution

5.1.2.1 Dynamic Programming Method

The foundation of this method is the principle of optimality, which may be stated in the following form:

> An optimal strategy has the property that whatever the initial state and the initial decisions are, the remaining decisions must constitute an optimal strategy with regard to the state resulting from the first decision.

The major shortcoming of this method is the large memory requirements that arise in cases of large scale systems. Also, dynamic programming has difficulty in dealing with multireservoir problems which have nonseparable benefits. Moreover, publications which have dealt with numerous dynamic programming models with stochastic inflows, have ignored the nonseparable benefits of the problem because of the large memory requirements of the dynamic programming method and the unavoidable calculations required in the total enumeration of states and allowable actions.

5.1.2.2 Linear, Quadratic, and Nonlinear Programming Methods

The problem of linear, quadratic, and nonlinear programming can be described by

$$\text{maximize } f(y) \tag{5.14}$$

$$\text{s.t. } Ay = b \tag{5.15}$$

$$\underline{y} \leq y \leq \overline{y} \tag{5.16}$$

where y is a vector of release and storage decision variables. The linear system of equations (5.15) is a set of flow conservation constraints, with the vector b as a set of exogenous inflows. The inequality constraints (5.16) place bounds on the flows to serve: flood control, navigational, fishing, water quality, recreational and other requirements. It should be pointed out that fundamentally different models arise from different choices of the objective function.

The main drawback of this method is the difficulty in handling inequality constraints. Also, there is no way to differentiate between hard and soft constraints using this method, and the only reasonable way to cope with this problem appears to be by the use of different weighting (Penalty) functions. Some techniques also require a feasible starting point [61].

Penalty functions

A penalty term reflecting the constraint violations is multiplied by a scalar weight and augmented to the actual performance index. If the augmented performance index is minimized by a sequence of increasing penalty weights, the solution of the successive unconstrained problems approaches the constrained solution. The main problem in using penalty functions is that there is no method to aid in choosing the sequence of penalty weights.

5.1.2.3 Discrete Maximum Principle Method

The discrete maximum principle method has not been a popular approach to the problem of multireservoir operation. This is because of numerical difficulties in solving the resulting two-point boundary value problem, especially when state variable constraints are included in the problem formulation.

5.1.2.4 Functional Analysis and Minimum Norm Formulation Method

The minimum norm formulation in the framework of functional analysis has been used successfully in the problem of optimal hydro scheduling. A typical feature of the approach is that it can yield necessary and sufficient conditions for the existence of solutions. This fact makes it possible to study the qualitative aspects of optimal processes. Moreover, this approach is free of the concrete nature of the system. More details on the method have been given in Chapter II.

5.1.3 Deterministic Versus Stochastic Solutions

The deterministic equivalent methods assume that the inflows are known during the whole planning period. In this way, instead of an operation strategy that produces the optimal operating decision U_t^* for each possible state x_t, it is enough to determine a trajectory $[x_1^*, x_2^*, \ldots, x_t^*, \ldots, x_T^*]$ which corresponds to the optimal reservoir evolution for the preestablished inflow sequence.

With the hypothesis of deterministic inflows, the dimensionality problem disappears. Although the remaining problem is still very complex, several algorithms are already available to solve it, such as those suggested by Rosenthal [49], Hanscom et al. [63], Murray and

Yokowitz [67], Ikura and Gross [68], and Gagnon et al. [7,9,50,61].

The theoretical basis for using deterministic inflows is the certainty equivalence principle, which establishes the optimal strategy for the solution of certain classes of stochastic control problems. This can be obtained by replacing the stochastic components by their expected values. It should be noted that this method assumes that the deterministic equivalent problem is resolved at each stage as soon as the new inflow measures become available.

The main advantage of deterministic equivalent methods is that they allow for a correct representation of the hydroelectric system. The main disadvantage of the methods is that they produce an optimistic operation which, in case of severe droughts, can lead to severe economic losses.

5.2 Quadratic Representation of the WCF

The generation of a hydro plant is a function of both its flow and its water head, i.e., the difference between its forebay and tailwater elevations, Fig. 2. The tailwater elevation is a function of the plant flow and of the storage of the reservoir immediately downstream. In our system, this last influence is negligible due to the distance from a plant to the following reservoir.

The generation of hydro-plants that have more than one generating unit cannot be assumed identical for most of the plants in the system. The generation of a unit, as a function of its flow, is zero up to a point q_1; then it increases up to the maximum generation flow q_3; any flow above q_3 is spillage which causes a decrease in generation due to an increase in tailwater elevation as in Fig. 3. The operating rule for the plants usually is, as the flow increases start the next unit when the generation rate of the previously started unit becomes too small. This is a little before the unit flow reaches q_3. In practice, the system operator must avoid operating the system of the intersection points since these points have the lowest generating efficiency.

Piecewise approximation usually guarantees a very close analogy to the original curves. The main drawback of using this method is that the derivative obtained from this piecewise approximation curve is highly non-continuous which may cause some difficulties during the optimization process. Piecewise linear approximation, Fig. 11 gives a reasonable approximation of the system. In this method the curve is approximated by a series of linear segments. To calculate the corresponding generation for a given discharge $Q_{k,i}$ first let

$$Q_{k,i} = \sum_{\ell=1}^{L} q_\ell \tag{5.17}$$

where q_ℓ is a component of $Q_{k,i}$ corresponding to a certain line segment of the linear approximating power production curve. As an example the value of Q on figure 11

$$Q_{k,i} = q_1 + q_2 + a\, q_3 \ , \ a = \text{constant} < 1.0$$

then the total power generated due to $Q_{k,i}$ is

$$P_{k,i} = \sum_{\ell=1}^{L} \eta_\ell \cdot q_\ell \tag{5.18}$$

The method recommended in this work calculates the corresponding

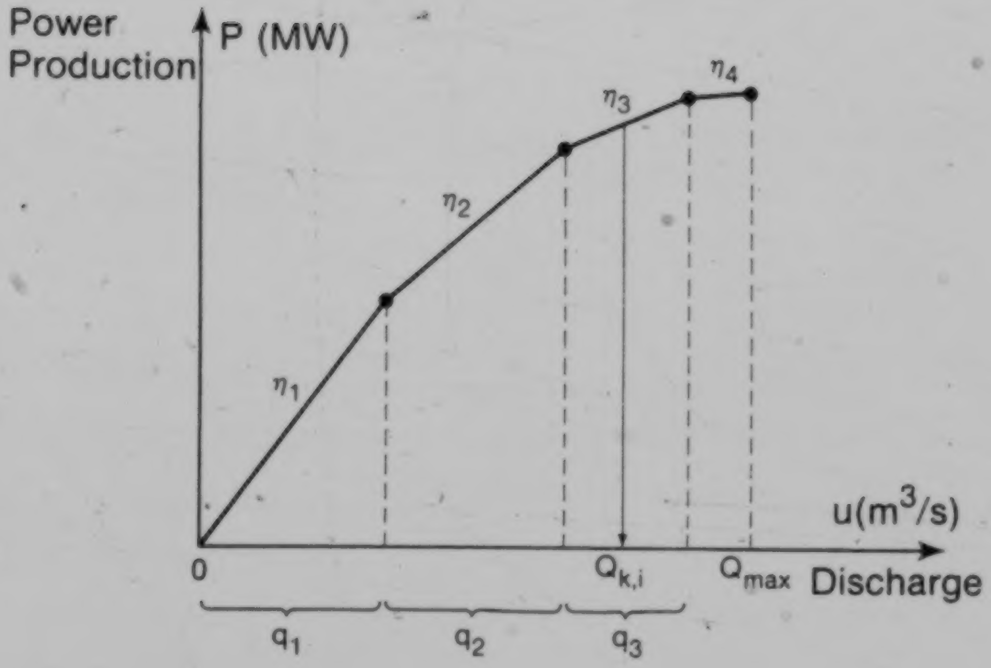

Fig. 11. Piecewise linear approximation of hydro production curve for a constant head (L=4).

value of the tailwater elevation during each time period for each plant. Then, the effective head is calculated by subtracting tailwater elevation from the forebay elevation of that period (function of the storage). Finally, both the effective head and the discharge through the turbine are used to calculate the total generation for each plant during each period. The method can be summarized as:

For the storage project, the total generation $G_{k,i}$ for plant k during a period i is

$$G_{k,i}=U_{k,i}\cdot[\alpha_{k,i}+\beta_{k,i}/2\cdot(X_{k,i}+X_{k,i-1})+\gamma_{k,i}/4\cdot(X_{k,i}+X_{k,i-1})^2] \tag{5.19}$$

To find the values of α_k, β_k, and γ_k for a reservoir k during a period i, the following steps are performed:

(a) calculate the optimal value of the storage $X_{k,i}$, the discharge through the turbine $U_{k,i}$ and the corresponding spillage $S_{k,i}$,

(b) from the table of total discharge versus tailwater elevation (U_k+S_k vs. TWE_k) calculate the tailwater elevation $TWE_{k,i}$ using a suitable cubic spline subroutine (Appendix I),

(c) construct a new table for the relation between the storage and the head (X_k vs. H_k)$_i$ using the field-measured table of the storage versus forebay elevation (X_k vs E_k) and the calculated $TWE_{k,i}$; $H_k=E_k-TWE_k$,

(d) match the (X_k vs H_k) table with the field-measured table of the WCF versus the head (WCF_k vs H_k) using a suitable cubic spline subroutine to conclude with a new table of the storage versus the WCF (X_k vs WCF_k)$_i$,

(e) calculate $\alpha_{k,i}$, $\beta_{k,i}$, and $\gamma_{k,i}$ for the $(X_k$ vs. $WCF_k)_i$ table using a least square curve fitting subroutine.

During this process, more weight is given to the points in each table that completely satisfy the boundary constraints. Using the average in Eq. (5.19) helps to avoid underestimation of the corresponding generation during rising water levels and overestimation during falling water levels.

In the previous chapter linear relationships between the WCF and the storage and elevation were used. This representation, while it is much more accurate than using constant WCF independent of the storage variation and head, still yields a large error in the calculated WCF. In this chapter, the over-all error considerably decreases as a result of using a quadratic relationship between the WCF and the storage and elevation, and also due to the consideration of a variable tailwater elevation rather than a constant one. Table 13 contains a comparison of the over-all maximum error in calculating the energy generation for some reservoirs from the B.P.A. hydro-system considering the quadratic, the third order, and the fourth order relationships between WCF and storage, and a cubic spline relationship between storage and forebay elevation with variable tailwater elevation.

Table 13

A comparison between the overall maximum error in the cases of quadratic, third order, and fourth order relationships between WCF and storage and a cubic spline relationship between storage and forebay elevation with variable tailwater elevation

Plant		Max.Storage Capacity KSFD	% Max.Error		
No.	Name		Quadratic	3rd Order	4th order
2/48	Mossyrock	654.3	1.505	0.996	-0.485
7/82	Swift I	225.4	0.256	0.024	0.024
14/173	Detroit	162.0	2.023	-0.435	-0.375
15/188	Foster	14.3	1.801	-0.8712	0.860
16/190	Green Peter	157.8	2.867	0.762	0.238
19/234	Cougar	77.4	2.428	1.099	-0.631
23/275	Lookout Pt.	169.7	1.997	-1.840	-1.326
24/290	Hills Crk.	122.8	2.783	1.847	-0.104
29/390	Round Butte	138.3	0.1927	0.213	0.168
39/767	Brownlee	494.2	0.530	-0.855	0.687
44/1210	Chelan	341.5	-0.067	-0.000	0.000
49/1305	Long Lake	52.5	-0.547	0.620	0.528
63/1510	Kerr	614.7	0.818	-0.306	-0.129
76/2025	Lwr. Baker	71.8	-0.437	-0.362	-0.360
80/2070	Ross	530.5	2.302	-0.874	-0.890
83/2190	Alder	84.0	1.977	0.512	-0.535
85/2208	Cushman I	187.6	1.108	0.418	0.137

5.3 Problem Formulation

The objective is to maximize the total generation for the whole system over the optimization horizon subject to satisfying the following conditions:

(1) The plant k expected energy generation during a time interval i, will be taken as:

$$G_{k,i} = E[WCF_{k,i} \cdot U_{k,i}] \tag{5.20}$$

where $WCF_{k,i}$ and $U_{k,i}$ are the WCF, watt hour per cubic feet, and the total discharge through the turbine of plant k during the i-th period respectively. The symbol E stands for the expected value.

(2) The water conservation equation of each reservoir may be adequately described by the continuity-type difference equation as:

$$X_{k,i} = X_{k,i-1} + INF_{k,i} + \sum_{r\in R_k} U_{r,i} - U_{k,i} + \sum_{r\in R_k} S_{r,i} - S_{k,i} \tag{5.21}$$

where

R_k is the set of plants immediately upstream of plant k,

$X_{k,i}$ is the storage of reservoir k at the end of the period i,

$INF_{k,i}$ is the total expected natural inflow after the deduction of the expected evaporation and percolation losses. The expected natural inflows are assumed to be statistically independent random variables, and

$S_{k,i}$ is the total spill from reservoir k during a period i.

(3) The WCF for a reservoir is modelled as a quadratic function of the

average storage, i.e.:

$$WCF_{k,i} = \alpha_{k,i} + \beta_{k,i/2} \cdot (X_{k,i} + X_{k,i-1}) + \gamma_{k,i/4}(X_{k,i} + X_{k,i-1}) \tag{5.22}$$

$\alpha_{k,i}$, $\beta_{k,i}$, and $\gamma_{k,i}$ are functions of the tailwater elevation (section 5.2, this chapter).

(4) The WCF for run-of-river (ROR) plants is considered as a constant.

(5) To satisfy the constraints that reflect physical limits, bank erosion, coordination agreements among various ownerships and multipurpose requirements such as irrigation, navigation, fishing, flood control, water quality, recreation, and other purposes if any, the plant variables must satisfy the following inequality (hard constraints):

(a) upper and lower bounds on reservoir contents,

$$\underline{X}_{k,i} \leq X_{k,i} \leq \overline{X}_{k,i} \tag{5.23}$$

where $\underline{X}_{k,i}$ and $\overline{X}_{k,i}$ are defined to satisfy physical limits and coordination agreement constraints on reservoir contents,

(b) upper and lower bounds on storage and ROR plant outflow,

$$\underline{U}_{k,i} \leq U_{k,i} \leq \overline{U}_{k,i} \tag{5.24}$$

where $\underline{U}_{k,i}$ and $\overline{U}_{k,i}$ are defined to satisfy system safety, the multipurpose stream use requirements and coordination agreements,

(c) maximum draft constraints to prevent any excessive soil erosion around the reservoir,

$$E_{k,i-1} - E_{k,i} \leq \overline{D}_k \qquad (5.25)$$

where $E_{k,i}$ is the forebay elevation of reservoir k at the end of a period i,

$\overline{D}_k$ is the maximum draft allowable for reservoir k.

Here, we redefine this equation to be an equation of the storage rather than the elevation. This is done by equating forebay elevation with a quadratic function of the storage, i.e.,

$$E_{k,i} = \phi_k + \psi_k \cdot X_{k,i} + \xi_k \cdot X_{k,i}^{2} \qquad (5.26)$$

ϕ_k, ψ_k, and ξ_k are constants determined by using the forebay elevation versus the storage (E_k vs X_k) table and a least squares curve fitting subroutine.

Substituting from Eq. (5.26) into Eq. (5.25) we get

$$\psi_k \cdot (X_{k,i-1} - X_{k,i}) + \xi_k \cdot (X_{k,i-1}^{2} - X_{k,i}^{2}) \leq \overline{D}_k,$$

$$i = 1,\ldots,m \text{ and } k=1,\ldots,n \qquad (5.27)$$

Equation (5.27) provides for maximum draft constraints and is a nonlinear state dependent inequality constraint.

(6) Each station is reduced to a single equivalent input/output curve to reduce the number of variables in the optimization process.

(7) The storage plants dictate how the immediate down stream ROR

plants operate since for the ROR plants, the outflow is equal to the total inflows (total inflows = total at site inflow + total outflow and spillage from upstream plants).

(8) The spillage usually causes a negative generation since it raises the tailwater elevation which eventually decreases the effective net head. The net effect is dependent on the design of the hydro-plant.

(9) The results of the algorithm will be used to allocate the half-monthly generation targets for each plant in the hydro-system. These targets will then be further refined into daily and finally hourly targets in real-time operation (a short-term study).

In summary, the objective is to maximize

$$J = E\Big[\sum_{k=1}^{n} \sum_{i=1}^{n} \big[\alpha_{k,i} \cdot U_{k,i} + 1/2 . \beta_{k,i} \cdot U_{k,i} \cdot (X_{k,i} + X_{k,i-1})$$

$$+ 1/4 . \gamma_{k,i} \cdot U_{k,i} \cdot (X_{k,i} + X_{k,i-1})^2\big]\Big] \qquad (5.28)$$

Subject to satisfying the equality constraints of Eq. (5.21), and the inequality constraints of Eqs. (5.23), (5.24) and (5.27); and taking into account the different relationships between the variables which have been given through the field measured tables.

5.4 **Minimum Norm Formulation**

The augmented cost function, $\hat{J}$, is obtained by adjoining to the cost function in Eq. (5.28) the equality constraints, Eq. (5.21), via Lagrange multipliers and the inequality constraints, Eqs. (5.23), (5.24) and (5.27), via Kuhn-Tucker multipliers. This yields

$$
\begin{aligned}
\hat{J}=E[\sum_{k=1}^{n} \sum_{i=1}^{m} [\alpha_{k,i}\cdot U_{k,i}+1/2.\beta_{k,i}\cdot U_{k,i}\cdot(X_{k,i}+X_{k,i-1})\\
+ 1/4.\gamma_{k,i}\cdot U_{k,i}\cdot(X_{k,i}+X_{k,i-1})^{2}\\
+ \lambda_{k,i}^{1}.(-X_{k,i}+X_{k,i-1}+INF_{k,i}+ \sum_{r\epsilon R_k} U_{r,i}-U_{k,i}\\
+ \sum_{r\epsilon R_k} S_{r,i}-S_{k,i})-e_{k,i}^{1}.(\underline{U}_{k,i}-U_{k,i})-e_{k,i}^{2}.(U_{k,i}-\bar{U}_{k,i})\\
- e_{k,i}^{3}.(\underline{X}_{k,i}-X_{k,i})-e_{k,i}^{4}.(X_{k,i}-\bar{X}_{k,i})\\
- \sigma_{k,i}\cdot[\psi_{k}\cdot(X_{k,i-1}-X_{k,i})+\varepsilon_{k}\cdot(X_{k,i-1}^{2}-X_{k,i}^{2})-\bar{D}_{k}]]]
\end{aligned}
\tag{5.29}
$$

In the above equation $\lambda_{k,i}^{1}$ is the Lagrange multiplier for plant k at period i, $e_{k,i}^{1}$, $e_{k,i}^{2}$, $e_{k,i}^{3}$, $e_{k,i}^{4}$, and $\sigma_{k,i}$ are Kuhn-Tucker multipliers. They are equal to zero if the corresponding constraints are not violated and greater than zero if the constraints are violated.

Substituting from Eq. (5.21) into Eq. (5.29), gives

$$
\begin{aligned}
\hat{J}=E[\sum_{k=1}^{n} \sum_{i=1}^{m} \{\alpha_{k,i}\cdot U_{k,i}+1/2.\beta_{k,i}\cdot U_{k,i}\cdot(2.X_{k,i-1}+ \sum_{R\epsilon R_k} U_{r,i}\\
- U_{k,i}+INF_{k,i}+ \sum_{r\epsilon R_k} S_{r,i}-S_{k,i})+1/4.\gamma_{k,i}\cdot U_{k,i}\cdot(4.X_{k,i-1}^{2}\\
+ [\sum_{r\epsilon R_k} U_{r,i}-U_{k,i}]^{2}+INF_{k,i}^{2}+[\sum_{r\epsilon R_k} S_{r,i}-S_{k,i}]^{2}+4X_{k,i-1}\cdot\\
[\sum_{r\epsilon R_k} U_{r,i}-U_{k,i}]+4X_{k,i-1}\cdot[\sum_{r\epsilon R_k} S_{r,i}-S_{k,i}+INF_{k,i}]
\end{aligned}
$$

$$+2.[\sum_{R\epsilon R_k} U_{r,i}-U_{k,i}].[\sum_{r\epsilon R_k} S_{r,i}-S_{k,i}+INF_{k,i}]$$

$$+\ 2.INF_{k,i}\cdot[\sum_{r\epsilon R_k} S_{r,i}-S_{k,i}])+\lambda_{k,i}^{1}\cdot(-X_{k,i}+Z_{k,i=1}+INF_{k,i}$$

$$+\sum_{r\epsilon R_k} U_{r,i}-U_{k,i}+\sum_{r\epsilon R_k} S_{r,i}-S_{k,i})-e_{k,i}^{1}\cdot(\underline{U}_{k,i}-U_{k,i})$$

$$-\ e_{k,i}^{2}\cdot\ {}_{k,i}-\bar{U}_{k,i})-e_{k,i}^{3}\cdot(\underline{X}_{k,i}-X_{k,i})-e_{k,i}^{4}\cdot(X_{k,i}-\bar{X}_{k,i})$$

$$-\ \sigma_{k,i}\cdot[\psi_k\cdot(X_{k,i-1}-X_{k,i})+\xi_k\cdot(X_{k,i-1}^{2}-X_{k,i}^{2})-\bar{D}_k]]]$$

(5.30)

We also have that

$$\sum_{i=1}^{m} \lambda_{k,i}^{1}\cdot X_{k,i} = \lambda_{k,i}^{1}\cdot X_{k,i}+\lambda_{k,2}^{1}\cdot X_{k,2}+\ldots$$

$$+\ \lambda_{k,m}^{1}\cdot X_{k,m}+\lambda_{k,o}^{1}\cdot X_{k,o}-\lambda_{k,o}^{1}\cdot X_{k,o}$$

$$= \lambda_{k,k}\cdot X_{k,m}-\lambda_{k,o}^{1}\cdot X_{k,o}+\sum_{i=1}^{m} \lambda_{k,i-1}\cdot X_{k,i-1}$$

(5.31)

$$\sum_{i=1}^{m} e_{k,i}^{3}\cdot X_{k,i} = e_{k,m}^{3}\cdot X_{k,m}-e_{k,o}^{3}\cdot X_{k,o}+\sum_{i=1}^{m} e_{k,i-1}^{3}\cdot X_{k,i-1}$$

(5.32)

$$\sum_{i=1}^{m} e_{k,i}^{4}\cdot X_{k,i}=e_{k,m}^{4}\cdot X_{k,m}-e_{k,o}^{4}\cdot X_{k,o}+\sum_{i=1}^{m} e_{k,i-1}^{4}\cdot X_{k,i-1}$$

(5.33)

$$\sum_{i=1}^{m} \sigma_{k,i}\cdot X_{k,i} = \sigma_{k,m}\cdot X_{k,m}-\sigma_{k,o}\cdot X_{k,o}+\sum_{i=1}^{m} \sigma_{k,i-1}\cdot X_{k,i-1}$$

(5.34)

$$\sum_{i=1}^{m} \sigma_{k,i} \cdot X_{k,i}^{2} = \sigma_{k,m} \cdot X_{k,m}^{2} - \sigma_{k,o} \cdot X_{k,o}^{2} + \sum_{i=1}^{m} \sigma_{k,i} \cdot X_{k,i-1} \tag{5.35}$$

Substituting from equations (5.31) to (5.35) into Eq. (5.30), gives

$$\begin{aligned}
\hat{J} = E[\sum_{k=1}^{n} \sum_{i=1}^{m} & [b_{k,i} \cdot U_{k,i} + d_{k,i} \cdot U_{k,i} \cdot X_{k,i-1} \\
& + 1/2 . d_{k,i} \cdot U_{k,i} \cdot (\sum_{r \in R_k} U_{r,i} - U_{k,i}) \\
& + \gamma_{k,i} \cdot U_{k,i} \cdot X_{k,i-1} \cdot (\sum_{r \in R_k} U_{r,i} - U_{k,i}) \\
& + \gamma_{k,i} \cdot U_{k,i} \cdot X_{k,i-1}^{2} + 1/4 . \gamma_{k,i} \cdot U_{k,i} \cdot (\sum_{r \in R_k} U_{r,i} - U_{k,i})^{2} \\
& + (\lambda_{k,i} - \lambda_{i-1}^{1}) . X_{k,i-1} + \lambda_{k,i}^{1} (INF_{k,i} + \sum_{r \in R_k} S_{r,i} - S_{k,i}) \\
& + \lambda_{k,i}^{1} \cdot (\sum_{r \in R_k} U_{r,i} - U_{k,i}) + \theta_{k,i}^{1} \cdot U_{k,i} + \theta_{k,i-1}^{2} \cdot X_{k,i-1} \\
& + (\sigma_{k,i} - \sigma_{k,i-1}) . \psi_{k} \cdot X_{k,i-1} + (\sigma_{k,i} - \sigma_{k,i-1}) . \xi_{k} \cdot X_{k,i-1}^{2}] \\
& + \lambda_{k,o}^{1} \cdot X_{k,o} - \lambda_{k,m}^{1} \cdot X_{k,m} + \theta_{k,m}^{2} \cdot X_{k,m} - \theta_{k,o}^{2} \cdot X_{k,o} \\
& + \sigma_{k,o} \cdot \psi_{k} \cdot X_{k,o} - \sigma_{k,m} \cdot \psi_{k} \cdot X_{k,m} + \sigma_{k,o} \cdot \xi_{k} \cdot X_{k,o}^{2} \\
& - \sigma_{k,m} \cdot \xi_{k} \cdot X_{k,m}^{2}]
\end{aligned} \tag{5.36}$$

where

$$\theta_{k,i}^{1} = e_{k,i}^{1} - e_{k,i}^{2} \qquad (5.37)$$

$$\theta_{k,i}^{2} = e_{k,i}^{3} - e_{k,i}^{4} \qquad (5.38)$$

$$b_{k,i} = \alpha_{k,i} + 1/2.\beta_{k,i}.INF_{k,i} + 1/2\beta_{k,i}.(\sum_{r\epsilon R_k} S_{r,i} - S_{k,i})$$
$$+1/4.\gamma_{k,i}.(INF_{k,i}^{2} + [\sum_{r\epsilon R_k} S_{r,i} - S_{k,i}]^{2} + 2.INF_{k,i}.[\sum_{r\epsilon R_k} S_{r,i} - S_{k,i}]) \qquad (5.39)$$

and

$$d_{k,i} = \gamma_{k,i}.(INF_{k,i} + \sum_{r\epsilon R_k} S_{r,i} - S_{k,i}) + \beta_{k,i} \qquad (5.40)$$

Constant parts have been dropped from Eq. (5.36).

Defining the pseudo-state variables:

$$T_{k,i} = U_{k,i} .(\sum_{r\epsilon R_k} U_{r,i} - U_{k,i}) \qquad (5.41)$$

and

$$Z_{k,i} = X_{k,i-1}^{2} \qquad (5.42)$$

this is done to reduce the polynomial nonlinear equation, Eq. (5.36), to a quadratic equation. Doing this and adjoining the equality equations, Eqs. (5.41) and (5.42), via Lagrange multipliers $\lambda_{k,i}^{2}$ and $\lambda_{k,i}^{3}$, to guarantee a satisfaction of the corresponding equality equations yields:

$$
\begin{aligned}
\hat{J}=E[\sum_{k=1}^{n} \sum_{i=1}^{m} [& b_{k,i}.U_{k,i}+d_{k,i}.U_{k,i}.X_{k,i-1} \\
& + 1/2.d_{k,i}.U_{k,i}.(\sum_{r\epsilon R_k} U_{r,i}-U_{k,i}) \\
& + \gamma_{k,i}.X_{k,i-1}.T_{k,i}+\gamma_{k,i}.U_{k,i}.Z_{k,i} \\
& + 1/4.\gamma_{k,i}.T_{k,i}.(\sum_{r\epsilon R_k} U_{r,i}-U_{k,i})+(\lambda_{k,i}^{1}-\lambda_{k,i-1}^{1}).X_{k,i-1} \\
& + \lambda_{k,i}^{1}.(INF_{k,i}+ \sum_{r\epsilon R_k} S_{r,i}-S_{k,i})+\lambda_{k,i}^{1}.(\sum_{r\epsilon R_k} U_{r,i}-U_{k,i}) \\
& + \theta_{k,i}^{1}.U_{k,i}+\theta_{k,i-1}^{2}.X_{k,i-1}+(\sigma_{k,i}-\sigma_{k,i-1}).\psi_k.X_{k,i-1} \\
& + (\sigma_{k,i}-\sigma_{k,i-1}).\xi_k.X_{k,i-1}^{2}+\lambda_{k,i}^{2}.U_{k,i}.(\sum_{r\epsilon R_k} U_{r,i}-U_{k,i}) \\
& - \lambda_{k,i}^{2}.T_{k,i}+\lambda_{k,i}^{3}.X_{k,i-1}^{2}-\lambda_{k,i}^{3}.Z_{k,i}] \\
& + \lambda_{k,o}^{1}.X_{k,o}-\lambda_{k,m}^{1}.X_{k,m}+\theta_{k,m}^{2}.X_{k,m}-\theta_{k,o}^{2}.X_{k,o} \\
& + \sigma_{k,o}.\psi_k.X_{k,o}-\sigma_{k,m}.\psi_k.X_{k,m}+\sigma_{k,o}.\xi_k.X_{k,o}^{2} \\
& - \sigma_{k,m}.\xi_k.X_{k,m}^{2}]
\end{aligned}
\tag{5.43}
$$

Defining the following vectors

$$X(I) = col.(X_{1,i},X_{2,i},\dots,X_{k,i},\dots,X_{n,i}) \tag{5.44}$$

$$U(I) = col.(U_{1,i},U_{2,i},\dots,U_{k,i},\dots,U_{n,i}) \tag{5.45}$$

$$b(I) = \text{col.}(b_{1,i},b_{2,i},\ldots,b_{k,i},\ldots,b_{n,i}) \tag{5.46}$$

$$\lambda_1(I) = \text{col.}(\lambda_{1,i}{}^1,\lambda_{2,i}{}^1,\ldots,\lambda_{k,i}{}^1,\ldots,\lambda_{n,i}{}^1) \tag{5.47}$$

$$\lambda_2(I) = \text{col.}(\lambda_{1,i}{}^2,\lambda_{2,i}{}^2,\ldots,\lambda_{k,i}{}^2,\ldots,\lambda_{n,i}{}^2) \tag{5.48}$$

$$\lambda_3(I) = \text{col.}(\lambda_{1,i}{}^3,\lambda_{2,i}{}^3,\ldots,\lambda_{k,i}{}^3,\ldots,\lambda_{n,i}{}^3) \tag{5.49}$$

$$\theta_1(I) = \text{col.}(\theta_{1,i}{}^1,\theta_{2,i}{}^1,\ldots,\theta_{k,i}{}^1,\ldots,\theta_{n,i}{}^1) \tag{5.50}$$

$$\theta_2(I) = \text{col.}(\theta_{1,i}{}^2,\theta_{2,i}{}^2,\ldots,\theta_{k,i}{}^2,\ldots,\theta_{n,i}{}^2) \tag{5.51}$$

$$\sigma(I) = \text{col.}(\sigma_{1,i},\sigma_{2,i},\ldots,\sigma_{k,i},\ldots,\sigma_{n,i}) \tag{5.52}$$

$$T(I) = \text{col.}(T_{1,i},T_{2,i},\ldots,T_{k,i},\ldots,T_{n,i}) \tag{5.53}$$

$$Z(I() = \text{col.}(Z_{1,i},Z_{2,i},\ldots,Z_{k,i}\ldots,Z_{n,i}) \tag{5.54}$$

$$\alpha(I) = \text{col.}(\alpha_{1,i},\alpha_{2,i},\ldots,\alpha_{k,i},\ldots,\alpha_{n,i}) \tag{5.55}$$

$$\beta(I) = \text{col.}(\beta_{1,i},\beta_{2,i},\ldots,\beta_{k,i},\ldots,\beta_{n,i}) \tag{5.56}$$

$$S(I) = \text{col.}(S_{1,i},S_{2,i},\ldots,S_{k,i},\ldots,S_{n,i}) \tag{5.57}$$

$$INF(I) = \text{col.}(INF_{1,i},INF_{2,i},\ldots,INF_{k,i},\ldots,INF_{n,i}) \tag{5.58}$$

$$e_1(I) = \text{col.}(e_{1,i}{}^1,e_{2,i}{}^1,\ldots,e_{k,i}{}^1,\ldots,e_{n,i}{}^1) \tag{5.59}$$

$$e_2(I) = \text{col.}(e_{1,i}^{\;2}, e_{2,i}^{\;2}, \ldots, e_{k,i}^{\;2}, \ldots, e_{n,i}^{\;2}) \tag{5.60}$$

$$e_3(I) = \text{col.}(e_{1,i}^{\;3}, e_{2,i}^{\;3}, \ldots, e_{k,i}^{\;3}, \ldots, e_{n,i}^{\;3}) \tag{5.61}$$

$$e_4(I) = \text{col.}(e_{1,i}^{\;4}, e_{2,i}^{\;4}, \ldots, e_{k,i}^{\;4}, \ldots, e_{n,i}^{\;4}) \tag{5.62}$$

Also, defining the following square matrices

$$d(I) = \text{diag.}(d_{1,i}, d_{2,i}, \ldots, d_{k,i}, \ldots, d_{n,i}) \tag{5.63}$$

$$\gamma(I) = \text{diag.}(\gamma_{1,i}, \gamma_{2,i}, \ldots, \gamma_{k,i}, \ldots, \gamma_{n,i}) \tag{5.64}$$

$$\psi = \text{diag.}(\psi_1, \psi_2, \ldots, \psi_k, \ldots, \psi_n) \tag{5.65}$$

$$\xi = \text{diag.}(\xi_1, \xi_2, \ldots, \xi_k, \ldots, \xi_n) \tag{5.66}$$

M = nxn matrix where the diagonal elements are equal to (-1), and the other elements vary between (1) and (zero) depending on the topological arrangement of the reservoirs and the ROR plants (see the example given in Chapter IV).

Then equation (5.43) can be rewritten as

$$\begin{aligned}\hat{J} = E\Big[\sum_{I=1}^{m} \{ & b^T(I).U(I) + U^T(I).d(I).X(I-1) \\ & + 1/2.U^T(I).d(I).M.U(I) + X^T(I-1).\gamma(I).T(I) \\ & + U^T(I).\gamma(I).Z(I) + 1/4.T^T(I).\gamma(I).M.U(I)\end{aligned}$$

$$+[\lambda_1^T(I)-\lambda_1^T(I-1)].X(I-1)+\lambda_1^T(I).[INF(I)+M.S(I)]$$

$$+\lambda_1^T(I).M.U(I)+\theta_1^T.U(I)+\theta_2^T(I-1).X(I-1)$$

$$+[\sigma^T(I)-\sigma^T(I-1)].\psi.X(I-1)+[\sigma^T(I)-\sigma^T(I-1)].\xi.$$

$$X(I-1).\vec{H}.X(I-1)+\lambda_2^T(I).U(I).\vec{H}.M.U(I)$$

$$-\lambda_2^T(I).T(I)+\lambda_3^T(I).X(I-1).\vec{H}.X(I-1)-\lambda_3^T(I).Z(I)]$$

$$+\lambda_1^T(o).X(o)-\lambda_1^T(m).X(m)+\theta_2^T(m).X(m)$$

$$-\theta_2^T(o).X(o)+\sigma^T(o).\psi.X(o)-\sigma^T(m).\psi.X(m)$$

$$+\sigma^T(o).\xi.X(o).\vec{H}.X.(o)-\sigma^T(m).\xi.X(m).\vec{H}.X(m)] \qquad (5.67)$$

$\vec{H}$ in the above equation is a vector nxn matrix in which the vector index varies from 1 to n. As an example let n=3

-vector index =1

$$H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

-vector index = 2

$$H_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

-vector index = 3

$$H_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and, given the two vectors

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

then

$$a.\vec{H}.c = \begin{bmatrix} a_1c_1 \\ a_2c_2 \\ a_3c_3 \end{bmatrix}$$

Defining the following vectors

$$W^T(I)=[X^T(I-1) \quad U^T(I) \quad T^T(I) \quad Z^T(I)] \tag{5.68}$$

$$R(I)=\begin{bmatrix} \lambda_1(I)-\lambda_1(I-1)+\theta_2(I-1)+[\sigma(I)-\sigma(I-1)].\psi \\ b(I)+M^T.\lambda_1(I)+\theta_1(I) \\ -\lambda_2(I) \\ -\lambda_3(I) \end{bmatrix} \qquad (5.69)$$

$$L(I)=\begin{bmatrix} \begin{array}{l}[\sigma(I)-\sigma(I-1)].\vec{H}.\xi \\ \lambda_3(I).\vec{H}\end{array} & 1/2.d(I) & 1/2.\gamma(I) & 0 \\ 1/2.d(I) & \begin{array}{l}1/4d(I).M+1/4.M^Td(I) \\ +1/2\gamma_2(I).\vec{H}.M \\ +1/2M^T.\vec{H}.\lambda_2(I)\end{array} & 1/8.M^T.\gamma(I) & 1/2\gamma(I) \\ 1/2.\gamma(I) & 1/8.\gamma(I).M & 0 & 0 \\ 0 & 1/2.\gamma(I) & 0 & 0 \end{bmatrix}$$

$$(5.70)$$

gives

$$\begin{aligned} \hat{J}=E[&-\lambda_1{}^T(m).X(m)+\theta_2{}^T(m).X(m)-\sigma^T(m).\psi.X(m) \\ &-\sigma^T(m).\xi.X(m).\vec{H}.X(m) \\ &+\sum_{I=1}^{m}[W^T(I).L(I).W(I)+R^T(I).W(I)]] \end{aligned} \qquad (5.71)$$

Constant terms are dropped from equation (5.71).

Eq. (5.71) is composed of a boundary part and a discrete integral part. The two parts are independent of each other. So, Eq. (5.71) can be written as

$$\hat{J} = \hat{J}_1 + \hat{J}_2 \tag{5.72}$$

where

$$\hat{J}_1 = E[-\lambda_1^T(m).X(m)+\theta_2^T(m).X(m)-\sigma^T(m).\psi.X(m)$$

$$-\sigma^T(m)\xi.X(m).\overset{+}{H}.X(m)] \tag{5.73}$$

$$\hat{J}_2 = E[\sum_{I=1}^{m} [W^T(I).L(I).W(I)+R^T(I).W(I)]] \tag{5.74}$$

If the vector $V(I)$ is defined such that

$$V(I) = L^{-1}(I).R(I) \tag{5.75}$$

then, the augmented cost function, $\hat{J}_2$, can be written as

$$\hat{J}_2 = E[\sum_{I=1}^{m} [[W(I)+1/2.V(I)]^T.L(I).[W(I)+1/2V(I)]$$

$$- 1/4.V(I).L(I).V(I)]] \tag{5.76}$$

$V(I)$ and $L(I)$ are independent of the variable $W(I)$. Dropping the constant terms from Eq. (5.76), yields

$$\hat{J}_2=E[\sum_{I=1}^{m} [[W(I)+1/2.V(I)]^T.L(I).[W(I)+1/2V(I)]]] \qquad (5.77)$$

Equation (5.77) defines a norm in Hilbert space. Hence Eq. (5.77) can be written as

$$\hat{J}_2=||W(I)+1/2V(I)||_{L(I)} \qquad (5.78)$$

5.5 The Optimal Solution

To maximize $\hat{J}$ in Eq. (5.72), we maximize each term separately [3], i.e.,

$$\underset{[X(m),W(m)]}{\text{Max}.\hat{J}} = \underset{[X(m)]}{\text{Max}.\hat{J}_1} + \underset{W(m)}{\text{Max}.\hat{J}_2}$$

The maximum of $\hat{J}_1$ is achieved when

$$\theta_2(m)-\lambda_1(m)-\sigma(m).\psi-\sigma(I).\xi.X(m)=\underline{0} \qquad (5.79)$$

Equation (5.79) gives the value of Lagrange multipliers as a function of the Kuhn-Tucker multipliers and the storage at the last period.

The maximum of Eq. (5.79) is mathematically equivalent to minimizing the norm of the same equation. The minimum of Eq. (5.78) is achieved when the norm of this equation is equal to zero; therefore,

$$E[W(I)+1/2V(I)] = \underline{0} \qquad (5.80)$$

Substituting from Eq. (5.75) into (5.80), gives

$$E[R(I)+2.L(I).W(I) = \underline{0} \qquad (5.81)$$

Equation (5.81) is the condition of optimality. Writing this equation explicitly and adding the equality constraints, produces

$$E[-\lambda_2(I)+\gamma(I).X(I-1)+1/4.\gamma(I).M.U(I)] = \underline{0} \qquad (5.82)$$

$$E[-\lambda_3(I)+\gamma(I).U(I)] = \underline{0} \qquad (5.83)$$

$$\begin{aligned} E[&b(I)+M^T.\lambda_1(I)+\theta_1(I)+d(I).X(I-1)+1/2.d(I).M.U(I) \\ &+1/2.M^T.d(I)+\lambda_2(I).\vec{H}.M.U.(I)+M^T.\vec{H}.\lambda_2(I).U(I) \\ &+1/4.M^T.\gamma(I).T(I)+\gamma(I).Z(I)] = \underline{0} \end{aligned} \qquad (5.84)$$

$$\begin{aligned} E[&\lambda_1(I)-\lambda_1(I-1)+\theta_2(I-1)+[\sigma(I)-\sigma(I-1)].\psi+2.\lambda_3(I).\vec{H}.X(I-1) \\ &+2.[\sigma(I)-\sigma(I-1)].\vec{H}.\xi.X(I-1)+d(I).U(I)+\gamma(I).T(I)] = \underline{0} \end{aligned} \qquad (5.85)$$

$$E[-X(I)-X(I-1)+INF(I)+M.U(I)+M.S(I)] = \underline{0} \qquad (5.86)$$

$$E[-T(I)+U^T(I).\vec{H}.M.U(I) = \underline{0} \qquad (5.87)$$

$$E[-Z(I)+X^T(I).\vec{H}.X(I)] = \underline{0} \qquad (5.88)$$

$$E[-b(I)+\alpha(I)+1/2.\beta(I).\vec{H}.\mathrm{INF}(I)+1/2.\beta(I).\vec{H}.M.S(I)$$

$$+1/4.\gamma(I).[\mathrm{INF}(I).H.\mathrm{INF}(I)+S(I).M^{T}.\vec{H}.M.S(I)$$

$$+2.\mathrm{INF}(I).\vec{H}.M.S(I)]] = \underline{0} \qquad (5.89)$$

$$E[-d(I)+\gamma(I).[\mathrm{INF}(I)+M.S(I)]+\beta(I) = \underline{0} \qquad (5.90)$$

$$E[-\theta_1(I)+e_1(I)-e_2(I)] = \underline{0} \qquad (5.91)$$

$$E[-\theta_2(I)+e_3(I)-e_4(I)] = \underline{0} \qquad (5.92)$$

Also, there are the following limits on Kuhn-Tucker multipliers

$$e_{k,i}^{1} = \begin{cases} 0.0 & \text{if } U_{k,i} \geq \underline{U}_{k,i} \\ >0.0 & \phantom{\text{if }} U_{k,i} < \underline{U}_{k,i} \end{cases} \qquad (5.93)$$

$$e_{k,i}^{2} = \begin{cases} 0.0 & \text{if } U_{k,i} \leq \overline{U}_{k,i} \\ >0.0 & \phantom{\text{if }} U_{k,i} > \overline{U}_{k,i} \end{cases} \qquad (5.94)$$

$$e_{k,i}^{3} = \begin{cases} 0.0 & \text{if } X_{k,i} \geq \underline{X}_{k,i} \\ >0.0 & \phantom{\text{if }} X_{k,i} < \underline{X}_{k,i} \end{cases} \qquad (5.95)$$

$$e_{k,14} = \begin{cases} 0.0 & \text{if } X_{k,i} \leq \bar{X}_{k,i} \\ >0.0 & \text{if } X_{k,i} > \bar{X}_{k,i} \end{cases} \tag{5.96}$$

$$\sigma_{k,i} = \begin{cases} 0.0 & \text{if } \psi_k \cdot (X_{k,i-1} - X_{k,i}) + \xi_k \cdot (X_{k,i-12} - X_{k,12}) \leq \bar{D}_k \\ >0.0 & \text{if } \psi_k \cdot (X_{k,i-1} - X_{k,i}) + \xi_k \cdot (X_{k,i-12} - X_{k,12}) > \bar{D}_k \end{cases} \tag{5.97}$$

Equations (5.82) to (5.97) with equation (5.79) completely specify the optimal solution.

5.6 Algorithm for Solution

Given: a system of n reservoirs, the expected values of natural inflows ($INF_{k,i}$ for each plant $k=1,\ldots,n$, at each period $i=1,\ldots,m$) the initial storage $X_{k,o}$ (all reservoirs must be full at the beginning of the critical period, i.e., $X_{k,o}=\bar{X}_{k,o}$, unless drafting for minimum flow or flood control); also, given the tables which specify the relationships between the variables for each plant at different periods and all the associated constraints:

(1) assume any initial value for $X_{k,i}$, $k=1,\ldots,n$ and $i=1,\ldots,m$

(2) let the spillage $S_{k,i}=0$, $k=1,\ldots,n$ and $i=1,\ldots,m$

(3) check the limits on $X_{k,i}$, so that

if $X_{k,i} < \underline{X}_{k,i}$ let $X_{k,i} = \underline{X}_{k,i}$

if $X_{k,i} > \bar{X}_{k,i}$ let $X_{k,i} = \bar{X}_{k,i}$

(4) calculate $U_{k,i}$ starting from the end reservoir or ROR plant and then go to the next down-stream plant till the end of the branch and take one branch at a time until they have all been finished. Calculate $U_{k,i}$, use the water conservation equation:

$$U_{k,i}=X_{k,i-1}-X_{k,i}+\text{INF}_{k,i}+\sum_{r\varepsilon R_k}(U_{r,i}+S_{r,i})-S_{k,i}$$

(5) check the limits on $U_{k,i}$

if $U_{k,i} < \underline{U}_{k,i}$ let $U_{k,i}=\underline{U}_{k,i}$ and calculate the corresponding value of $X_{k,i}$ using the above conservation equation,

if $U_{k,i} > \overline{U}_{k,i}$ let $U_{k,i} = \overline{U}_{k,i}$ and calculate the corresponding value of $X_{k,i}$ using the above conservation equation,

(6) check the limits on $X_{k,i}$

if $X_{k,i} < \underline{X}_{k,i}$ a) let $X_{k,i} = \underline{X}_{k,i}$,
b) calculate the corresponding value of $U_{k,i}$,
c) let $e_{k,i}^{1} > 0.0$

if $X_{k,i} > \overline{X}_{k,i}$ a) let $X_{k,i} = \overline{X}_{k,i}$
b) let $U_{k,i} = \overline{U}_{k,i}$
c) calculate the corresponding value of $S_{k,i}$
d) let $e_{k,i}^{2} > 0.0$

Another alternative is to use the soft constraints $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$ in the preceding period to increase or decrease the discharge, so that searching for a suitable Kuhn-Tucker value can be avoided and the optimum solution can be reached much faster..

In the case when both $X_{k,i}$ and $U_{k,i}$ satisfy the boundary constraints then the corresponding Kuhn-Tucker multipliers will be set at zero.

(7) using the calculated values of $X_{k,i}$ and the values of ψ_k,ξ_k and $\bar{D}_k$ calculate the value of $\sigma_{k,i}$ from the following

if $\psi_k\cdot(X_{k,i-1}-X_{k,i})+\xi_k\cdot(X_{k,i-1}^2-X_{k,i}^2)\leq\bar{D}_k$ then $\sigma_{k,i}=0.0$

if $\psi_k\cdot(X_{k,i-1}-X_{k,i})+\xi_k\cdot(X_{k,i-1}^2-X_{k,i})>\bar{D}_k$ then $\sigma_{k,i} > 0.0$

(8) using the calculated values of Kuhn-Tucker multipliers at the last period m, $\theta_{k,m}^2$ and $\sigma_{k,m}$ and the constants ψ_k and ξ_k and the calculated storage at the last period $X_{k,m}$, calculate the value of $\lambda_{k,m}^1$ at the last period, m, using equation (5.79) and (5.92).

(9) calculate $\theta_{k,i}^I$, $T_{k,i}$, $Z_{k,i}$, $b_{k,i}$ and $d_{k,i}$ using equations (5.91), (5.87), (5.88), (5.89), and (5.90).

(10) calculate the tailwater elevation for each reservoir, then the effective head, and then calculate the values of the WCF constants $\alpha_{k,i}$, $\beta_{k,i}$, and $\gamma_{k,i}$,

(11) calculate $\lambda_{k,i}^3$ using Eq. (5.83),

(12) calculate $\lambda_{k,i}^2$ using Eq. (5.82),

(13) calculate $\lambda_{k,i}^1$ using the calculated values of $\lambda_{k,m}^1$ at last period (step 8) using the backward approach solution of equation (5.85),

(14) calculate a new updated value of $X_{k,i-1}$ using Eq. (5.84),

(15) calculate the value of the objective function and check if the solution converges toward a better solution, then continue (go to step

2) until no significant changes occur from iteration to iteration. If the solution diverges stop the calculation and then use the results to modify the controlling soft constraints $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$, described in Section 4.13, Chapter IV; and then repeat the calculation starting from step 1. Here we don't have to change the initial estimate from trial to trial.

5.7 Application to the B.P.A. System

The algorithm is used to solve the B.P.A. hydro-power system (37 reservoir type and 51 ROR type, see Chapter III, Section 3.3). The optimization is done in half-month periods for four years (96 periods) which include the 42 month (84 periods) critical period.

This time we have considered the generation from each reservoir as a quadratic function of the storage times the discharge through the turbine. Also, tailwater elevation is taken as a nonlinear function of the total discharge and calculated each time using cubic spline curve fitting. Moreover, the relation between the water content in each reservoir is generally a nonlinear function of the forebay elevation which is computed each time using cubic spline curve fitting. In this chapter, we also considered the maximum draft constraints (nonlinear state dependent inequality type of constraints), along with the other types of equality and inequality constraints (the linear type).

The B.P.A. hydro-system projects are varied. No two hydro-electric systems among the whole project are alike. The reasons for the differences among the plants are the natural differences in the watersheds, the differences in the man-made storage and release elements used to control water flows, and the very many different types of natural and man-made constraints imposed on the operation of the hydro-electric systems. The B.P.A. river system extends over vast multinational areas and includes many tributaries and complex arrangements of storage reservoirs.

Because of the nature of some projects, six projects have been grouped so that each two are considdered as if they are one project. This decreases the computational effort; and will not affect the distinctive features for any of the projects (Section 4.15, Chapter IV). Also, a special subroutine has been written to suit the individual topological configuration and characteristics of the Canal Plant and the Corra Linn projects, Fig. 9 and Table 126, Ref. 74.

The cubic spline technique is a good fit for all of the field measured tables in all the 88 projects except for two tables of the tailwater elevation versus the total discharge (discharge through the turbine plus the spillage) for The Yale (Section 3.3.5) and The Upper Baker (Section 3.3.77) projects, Fig.12.

The program takes 9.7 seconds of central processing unit time on the MTS-AMDAHL-5870 computer for each trial (3 iterations). I have performed 14 trials until I reached the global maximum for the proposed technique. After each trial I modify $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$ (Section 4.13) especially those which have been violated by the program, to satisfy the system hard constraints. The results show:

(1) no violation of any of the hard constraints,

(2) very small (unavoidable I believe) amount of spill compared to the results from the B.P.A. rule curves, and more importantly,

(3) an increase in the system total energy capability of 4.7% over that given when applying the B.P.A. rule curves.

The discretized system state and consequently, the computational

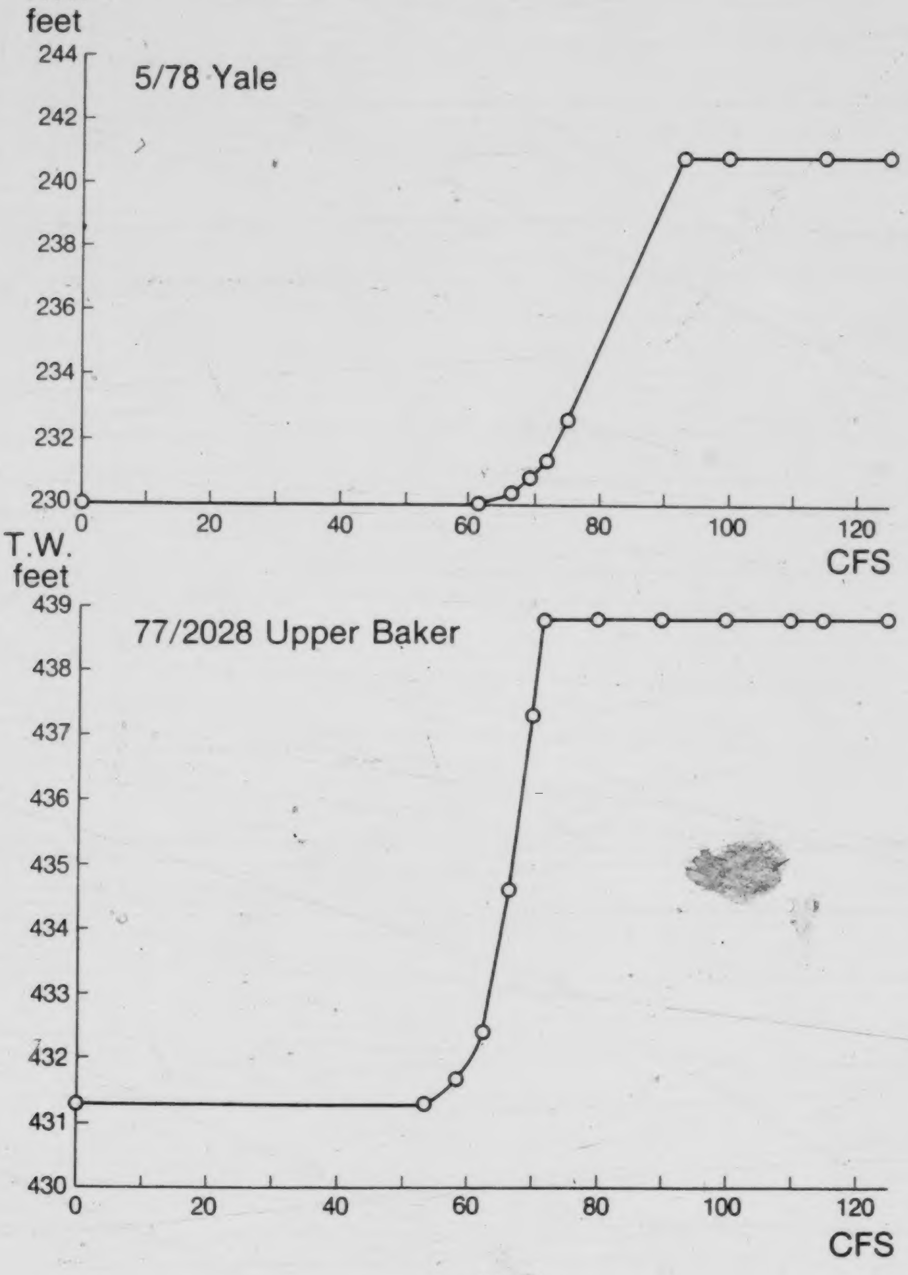

Fig. 12 Tailwater Variation Versus Total Discharge for Yale and Upper Baker Reservoirs.

effort, increases linearly with the number of state variables. This is a very important feature of this technique that enables it to deal with highly complicated problems. For example, in Pereira, 1985 [25], the number of the system states increases exponentially with the number of state variables.

The hard constraints (Section 3.3, Chapter III) have covered two essential types:

(1) constraints that enforce feasibility due to physical and/or technical features in the system, and

(2) constraints that guarantee a complete satisfaction of the contractual agreements and regulations related to flood control, wildlife, fisheries requirements, water quality, recreational use, etc. Table 14 gives a comparison between our results and the results from applying the B.P.A. rule-curves.

Table 14

Comparison between the energy capability measured when applying the results of our technique to those obtained from the B.P.A. rule curves

Period	Average Power in GW	
	B.P.A.	Calculated
1st 10 month critical period	12.373	12.594
next 12 month critical period	11.801	12.511
next 12 month critical period	12.321	12.211
next 8 month critical period	12.163	13.974
Total energy in GWH	372.512	398.891
Average power in GW	12.155	12.725
The % increase	4.7%	

5.8 Conclusions

A presentation has been made to illustrate the solution technique for the half-monthly operating policy of a multireservoir-tree connected hydro-electric power system. The technique is applied to the B.P.A. hydro system. The problem is believed to be one of the largest hydro-electric nonlinear problems attempted considering the number of variables and linear and nonlinear, equality and inequality constraints.

In this study the intent was to use a much more accurate model than the one used in Chapter IV. It is assumed here that the WCF is a quadratic function of the average storage and that the tailwater elevation is variable and depends on the total release (and is calculated using cubic spline curve fitting). Maximum draft specifications at various B.P.A. plants (which impose nonlinear inequality constraints on the technique) are taken into account in this task, and the relation between the storage and the forebay elevation is considered nonlinear and computed using cubic spline curve fitting.

The problem is formulated as a minimum norm problem in the framework of functional analysis optimization techniques. We formulate the problem by constructing a cost function in which the total generation of all the plants is maximized. To cast this cost function into quadratic form psesudo-state variables are defined. The augmented cost function is composed by adjoining all equality constraints to the original cost function via Lagrange multipliers and all the inequality constraints via Kuhn-Tucker multipliers. Finally, all terms of equal order are collected to form a norm and the solution is subsequently obtained by applying the minimum norm formulation technique.

The basic feature of this new procedure is its ability to automatically produce a maximum hydro-generation while satisfying constraints. The technique overcomes the influence of starting points and is able to combine methodology and experience to end with the system global maximum.

CHAPTER VI

Efficient Load Following Scheduling for Large-Scale Hydro System

Power systems have steadily grown in size and complexity. Small hydroelectric systems have been replaced by large and complicated networks with multiple fuel generation. This has guided the optimization techniques toward mixed fuel generation scheduling or considering other sources of generation by subtracting the value of these resources from the firm load.

This chapter will deal with the problem of scheduling the hydro-release to obtain the best operating policy for the system under study, where best means maximum and most uniform power surplus while satisfying the various environmental, physical, legal, and contractual constraints. The algorithm considers the generation from each reservoir as a quadratic function of the storage times the discharge through the turbine. Tailwater elevation is considered variable and a function of the total release. A nonlinear relationship between the storage and forebay elevation is assumed and solved using suitable cubic spline curve fitting. Maximum draft constraints have imposed a nonlinear state dependent inequality constraint type to the proposed technique.

Critical period (CP) optimization is used to help in determining the operation that maximizes the average CP generation, shaped uniformly to the load, while meeting hard constraints and balancing any violations of soft constraints. The model does this by varying the storage contents of all projects and all periods so as to meet the given load, or the "dependable hydrogeneration".

Dependable hydrogeneration is calculated by subtracting from the total firm load a nearly uniform amount representing the required thermal, nuclear, solar, wind, wave, and other resources in the system. The remaining, dependable load, must be met with the hydro generation under all historical streamflow conditions, this is called dependable hydrogeneration. The historical streamflow sequence which will produce only the dependable hydrogeneration is called the critical flow; the duration of this sequence is called the critical period. The rule-curve operation will never draw below these levels except to serve firm load so that no curtailment of firm load isnecessary unless a sequence worse than that of the historical record occurs.

The availability of limited amounts of hydroelectric energy in the form of stored water in the system reservoirs, makes the optimal operation problem very complex because it creates a link between an operating decision in a given stage and the future consequences of this decision. In other words, if we deplete the stocks of hydroelectric energy, and low inflow volumes occur, it may be necessary to use very expensive thermal generation in the future or even fail to supply the load. On the other hand, if we keep the reservoir levels high through a more intensive use of thermal generation and high inflow volumes occur, there may be spillage in the system, which means a waste of energy, and consequently, higher operating cost. So, the availability of the fuel, natural water inflow, must itself be forecasted.

6.1 **Background**

The hydro system scheduling problem (HSSP) is mainly to determine the water releases from the reservoirs and the flows through the power houses so as to maximize the values of the total energy generation and assure most uniform surplus subject to a number of constraints. The latter include environmentalconsiderations irrigational demands, flood control limitation, water quality requirement, fisheries, navigational demands, recreational use requirements, operational practices, contractual obligations and other water use legislation. The determination of optimal schedules entails the solution of a highly nonlinear problem while accommodating a group of linear and nonlinear, equality and inequality constraints.

A wide variety of techniques for solving such problems have been reported in the literature. Christensen, El-Hawary, and Soliman [1,2,70] and Rosenthal [60] have presented a comprehensive survey for the problem of multireservoir, multiperiod deterministic or stochastic inflow, and nonseparable benefit. In the literature, the solution of the problem has been done using linear [21] quadratic [26], nonlinear programming [7,9,47.50,54.68], or using dynamic programming [14,15,16.18,21,51,52]. The approach in [7] is to transform a general nonlinear program formulation of the HSSP into an optimization problem with a nonlinear objective and only linear constraints. All the nonlinear constraints are expressed as penalty terms and are added to the original objective function. This approach [7] is considered as the first successful attempt to solve a large HSSP with a nonlinear optimization technique [68]. However, the computational times reported are rather lengthy.

Another noteworthy application of the large scale nonlinear programming techniques is the methodology developed by Hanscom et al. [63]; a simpler model without any nonlinear constraints is employed and a large-scale scheduling problem is solved within less time than that taken by Hicks et al. [7].

Ikura and Gross [68] present a hydro scheduling problem formulation and a method of solution to this problem as a nonlinear programming problem. The maximum turbine discharge constraint is approximated by a piece-wise linear function. A nonlinear constraint for forced spill is treated as a penalty factor. The resulting problem has a nonlinear objective function with linear constraints and is solved using standard mathematical programming packages. A network flow algorithm is used to provide a good starting point for the standard programming packages. In these works [68] they have distinguished between forced and controllable spills. I disagree with them I think that the authors' proposed model of spill is unnecessarily complex. Practically, if the storage, $X_{k,i}$, computed with maximum possible controlled release exceeds the reservoir capacity, $\overline{X}_{k,i}$, all excess water, $X_{k,i}-\overline{X}_{k,i}$, will be released to avoid jeopardizing the dam. This operation policy can be modeled more simply than proposed by the authors. The total reservoir release can be defined as the sum of controlled and uncontrolled release. A large penalty can be assigned to uncontrolled release. This should force the uncontrolled release to be zero unless the upper bound reservoir storage would be violated.

Using this method [68], Ikura and Gross have found in some cases that the ratio between computational time required to solve their problem using a nonlinear programming technique associated with a scheme for determining a good starting point, to that without any scheme for determining a starting point is roughly 1:2. Thus, in cases of practical interest, the use of a suitable scheme to determine first a good starting point results in halving the computational time. In other words, the overall solution time in their method is reduced by about 50%, using a good starting point determined by the application of linear network flow algorithm.

The dynamic programming formulation [62] converts the HSSP with a high dimensional state space into a sequence of problems with lower dimensional state spaces. In almost all dynamic programming techniques, the computational requirements increase exponentially with the increase of the dimension of the problem. Murray and Yokowitz [69] have successfully run the HSSP with a memory and computational requirements that grow only as n^2 and n^3, respectively instead of exponentially with n.

The solution method used by Rosenthal [49] makes good use of the underlying network structure of the hydro system scheduling problem. The formulation has a general nonlinear objective with linear network flow constraints. The algorithm uses the solution of an integer programming problem to determine the search direction.

This chapter presents an efficient alternative based on functional analysis and the minimum norm formulation technique. The ability of this technique to produce the maximum energy capability with a uniform surplus power while completely satisfying all the system hard constraints is evident and the results show much promise.

6.2 Problem Description

The primary objective in the operation of a hydro electric generation system is to use the water in the most efficient way, while satisfying all constraints imposed by legal contractual obligations, physical characteristics and operating policies. The optimal water release schedules are determined at each reservoir so as to meet this goal. Due to the large-scale network structure of hydro systems of practical interest, determination of such schedules entails the solution of rather complex problems. In this chapter a stochastic model of the hydroelectric generation system and a computationally efficient method to solve the hydro system scheduling problem will be presented. The intended application of the scheduler is to the planning of the operation strategy over a specified period, the critical period.

The initial amount of storage in each reservoir and the expected natural inflows into each stream during each period are assumed to be known. All the hydro system reservoirs must be full at the beginning of the critical period unless drafting for minimum flow or flood control. The forecasts of natural inflows are obtained using historical rainfall, river measurements and snow survey data. The demand for water, typically for irrigational purposes, is also assumed to be known at each location in each subperiod. The travel time from one hydro facility to another is not taken into account since the transportation delays are very much smaller than the duration of each period (half a month).

In this chapter, the generation from each reservoir will be considered as a quadratic function of the storage times the release through power house. Tailwater elevation is considered variable and calculated each time knowing the total release and using typical field measured tables of tailwater elevation versus total release. The storage is considered as a nonlinear function of the elevation and accounted for using a suitable cubic spline curve fitting. All constraints of all types, linear , nonlinear, equality and inequality are considered. And then, a load following problem where the objective is to maximize the total energy capability of the system and to assure the most uniform surplus power to the system after satisfying the given load and all given hard constraints is presented.

The hydro system load following problem is formulated as a large-scale nonlinear problem and then solved using the minimum norm formulation in the framework of functional analysis.

6.3 Problem Formulation

The system under study consists of n reservoirs on different branches of a river in an arbitrary topological configuration, this represents the general case. The problem is to find the discharge $U_{k,i}$, i is the period number, i=1,...,m, and k is the project number, k=1,...,m subject to satisfying the following conditions:

(1) The total generation from the whole system over the optimization intervals is a maximum.

(2) The generation satisfies the given load (dependable load) during each period of the optimization horizon.

(3) Surplus power is as uniform as possible.

(4) The plant k expected energy generation during the time interval i depends on the head and the flow through the power house (an example of such a relation is shown in Fig. 4 . The generation will be taken as:

$$G_{k,i}=E[\alpha_{k,i}\cdot U_{k,i}+1/2\beta_{k,i}\cdot U_{k,i}\cdot(X_{k,i}+X_{k,i-1})$$

$$+\ 1/4\gamma_{k,i}\cdot U_{k,i}\cdot(X_{k,i}+X_{k,i-1})^2 \qquad (6.1)$$

$X_{k,i}$ is the storage of plant k at the end of a period i; $\alpha_{k,i}$, $\beta_{k,i}$ and

$\gamma_{k,i}$ are functions of the tailwater elevation. To calculate the values of $\alpha_{k,i}$, $\beta_{k,i}$, and $\gamma_{k,i}$ for a reservoir at a given period i:

(a) we calculate first the value of the storage $X_{k,i}$, the discharge through the turbine $U_{k,i}$ and the spill $S_{k,i}$,

(b) from the table of total discharge versus tailwa ter elevation (U_k+S_k vs TWE_k), we calculate the corresponding tailwater elevation $TWE_{k,i}$ using a suitable cubic spline subroutine, Appendix 1,

(c) we construct a new table between the storage and the head (X_k, vs H_k) using the field measured table of the storage versus forebay elevation (X_k vs E_k) and the calculated tailwater elevation; $H_k=E_k-TWE_{i,i}$,

(d) we match the (X_k vs H_k) table with the field-measured table of the water conversion factor versus the effective head (WCF_k vs H_k) using a suitable cubic spline subroutine to conclude with a new table of the storage versus the water conversion factor (X_k vs WCF_k),

(e) we calculate $\alpha_{k,i}$, $\beta_{k,i}$ and $\gamma_{k,i}$ for the (X_k vs WCF_k) table using a least squares curve fitting subroutine.

During these processes, more weight has been given to the points in each table which satisfy the boundary constraints.

Using the average in Eq. (6.1) helps to avoid underestimation in the corresponding generation during rising water levels and overestimation during falling water levels.

The symbol E stands for the expected value.

(5) The water conservation equation for each reservoir may be adequately described by the continuity-type difference equation as:

$$X_{k,i}=X_{k,i-1}+INF_{k,i}+\sum_{r\in R_k} U_{r,i}-U_{k,i}+\sum_{r\in R_k} S_{r,i}-S_{k,i} \tag{6.2}$$

R_k is the set of plants immediately upstream of plant k, $INF_{k,i}$ is the expected natural inflow for plant k during a period i. The expected natural inflows are considered as statistically independent random variables.

(6) To satisfy multi-purpose stream use requirements such as flood control navigation, irrigation, fishing, water quality, recreational activities, and other purposes if any, the plant variables must satisfy the following inequality constraints:

(a) upper and lower bounds on reservoir contents,

$$\underline{X}_{k,i} \leq X_{k,i} \leq \overline{X}_{k,i} \qquad (6.3)$$

$\overline{X}_{k,i}$ and $\underline{X}_{k,i}$ are defined to satisfy flood control constraints and physical limits on the reservoir contents.

(b) upper and lower bounds on reservoir and run-of-river plant outflow,

$$\underline{U}_{k,i} \leq U_{k,i} \leq \overline{U}_{k,i} \qquad (6.4)$$

$\overline{U}_{k,i}$ and $\underline{U}_{k,i}$ are defined to satisfy the system safety and the multipurpose stream use requirements and coordination agreements among various ownerships.

(c) maximum draft constraints to prevent any excessive soil erosion around the reservoir

$$E_{k,i-1} - E_{k,i} \leq \overline{D}_k \qquad (6.5)$$

$E_{k,i}$ is the forebay elevation of plant k at the end of a period i. $\overline{D}_k$ is the maximum draft requirement for plant k. Here, we reform this equation to be an equation of the storage rather than the forebay elevation. This is done by equating forebay elevation with a quadratic function of the storage

$$E_{k,i} = \phi_k + \psi_k \cdot X_{k,i} + \xi_k \cdot X_{k,i}^2 \qquad (6.6)$$

ϕ_k, ψ_k, and ξ_k are plant k constants calculated using the (E_k vs X_k) table and a least squares curve fitting subroutine.

Eq. (6.5) then can be written as

$$\psi_k \cdot (X_{k,i-1} - X_{k,i}) + \xi_k \cdot (X_{k,i-1}^2 - X_{k,i}^2) \leq \overline{D}_k \qquad (6.7)$$

Eq. (6.7) takes care of draft constraints and it is a nonlinear state dependent inequality constraint.

(7) Each station is reduced to a single equivalent input/output curve to reduce the number of variables in the optimization process.

(8) Storage plants dictate how the immediate down stream run-of-river plants operate.

(9) The spill usually causes a negative generation since it raises the tailwater elevation which eventually decreases the effective net head. The net effect is dependent on the design of the hydro-plant.

(10) The results of the algorithm can be used to allocate the half-monthly generation targets for each plant in the hydro-system. These targets are then further refined into daily and finally hourly targets in real-time operation.

6.4 Uniform Surplus Power

The objective is to determine the optimal release that maximizes the average critical period generation, shaped uniformly to the load, while meeting hard constraints and balancing the violation, if any, of soft constraints. There are many techniques that can handle this problem such as:

(1) Maximize $\sum_{i=1}^{m} \sum_{k=1}^{n} G_{k,i}$ (6.8)

subject to satisfying the equality and the inequality constraints and subject to satisfying the following inequality constraints

$$\sum_{k=1}^{n} G_{k,i} - \ell_i - C \geq 0.0, \qquad i=1,\ldots,m \qquad (6.9)$$

ℓ_i is the dependable load during a period i

C is a constant to represent the expected surplus energy and calculated each iteration so as to assure, if satisfied, the most uniform surplus power during the optimization horizon. In Eq. (6.9) inequality is preferred than the equality because the total surplus energy is variable from iteration to iteration so, it is diffulct to predict the value of C exactly. Also, due to the characteristics of the problem, and as we will see, when we apply the technique to a system in operation, that it is almost impossible to have a completely uniform surplus power due to many hard constraints which have to be satisfied first.

(2) Maximize

$$\sum_{i=1}^{m} \left[\sum_{k=1}^{n} G_{k,i} - \pi_i \left(\sum_{k=1}^{n} G_{k,i} - \ell_i - C \right)^2 \right] \qquad (6.10)$$

subject to satisfying the equality and inequality constraints. π_i here is a constant to represent a penalty factor during a period i. The value of π_i may have different values as the solution converges to the optimum value. The value of π_i should not take a large value because it may affect the main problem of maximizing the total energy production and also π_i should be greater than zero. From my experience

$$0.0<\pi_i<0.6 \tag{6.11}$$

(3) maximize

$$\sum_{i=1}^{m} \sum_{k=1}^{n} G_{k,i} \tag{6.12}$$

subject to satisfying the equality and inequality constraints and subject to satisfying the following inequality constraints:

$$\underline{U}_{k,i} \leq \hat{U}_{k,i} \leq \overline{U}_{k,i} \tag{6.13}$$

$$\underline{X}_{k,i} \leq \hat{X}_{k,i} \leq \overline{X}_{k,i} \tag{6.14}$$

Eqs. (6.13) and (6.14) will be handled as soft constraints as mentioned before in Chapter IV. Here, the value of $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$ will not only be determined according to the recommended ranges from the experience gained from dealing with the system, but also determined so that equation (6.9) will be verified.

(4) combination of methods (1) and (3)

I have worked through all these methods and I have found that method (4) is the best one among others that gives maximimum energy and the most uniform surplus power in an acceptable computing time.

In summary, the objective is to maximize

$$J = E(\sum_{k=1}^{n} \sum_{i=1}^{m} G_{k,i}) \qquad (6.15)$$

subject to satisfying the equality constraints of Eq. (6.2) and the inequality constraints of Eqs. (6.3), (6.4), (6.7), and (6.9); and to satisfying to the maximum extent the soft constraints of Eqs. (6.13) and (6.14). Also, to consider different relations of the variables that have been given through the field measured tables.

6.5 Minimum Norm Formulation

Substituting from Eq. (6.1) and (6.2) into Eq. (6.15), one obtains

$$\begin{aligned} J=E(\sum_{i=1}^{m} [& b_{k,i} \cdot U_{k,i} + d_{k,i} \cdot U_{k,i} \cdot X_{k,i-1} + 1/2 d_{k,i} \cdot U_{k,i} \cdot \\ & (\sum_{r \epsilon R_k} U_{r,i} - U_{k,i}) + \gamma_{k,i} \cdot X_{k,i} \cdot U_{k,i} \cdot (\sum_{r \epsilon R_k} U_{r,i} - U_{k,i}) \\ & + 1/4 \gamma_{k,i} \cdot U_{k,i} \cdot (\sum_{r \epsilon R_k} U_{r,i} - U_{k,i})^2 \\ & + \gamma_{k,i} \cdot U_{k,i} \cdot X_{k,i-1}^2]) \end{aligned} \qquad (6.16)$$

where

$$\begin{aligned} b_{k,i} = & \alpha_{k,i} + 1/2 \beta_{k,i} \cdot (INF_{k,i} + \sum_{r \epsilon R_k} S_{r,i} - S_{k,i}) \\ & + 1/4 \gamma_{k,i} \cdot [INF_{k,i}^2 + (\sum_{r \epsilon R_k} S_{r,i} - S_{k,i})^2 + 2.INF_{k,i} \cdot \\ & (\sum_{r \epsilon R_k} S_{r,i} - S_{k,i}) \end{aligned} \qquad (6.17)$$

$$d_{k,i}=\beta_{k,i}+\gamma_{k,i}\cdot(INF_{k,i}+\sum_{r\varepsilon R_k} S_{r,i}-S_{k,i}) \qquad (6.18)$$

If we define the following pseudo variables such that

$$Z_{k,i}=X_{k,i-1}^{2} \qquad (6.19)$$

and

$$T_{k,i}=U_{k,i}\cdot(\sum_{r\varepsilon R_k} U_{r,i}-U_{k,i}) \qquad (6.20)$$

Eq. (6.16) can be written as

$$J=E[\sum_{i=1}^{m}[b_{k,i}\cdot U_{k,i}+d_{k,i}\cdot U_{k,i}\cdot X_{k,i-1}+1/2d_{k,i}\cdot U_{k,i}\cdot$$

$$(\sum_{r\varepsilon R_k} U_{r,i}-U_{k,i})+\gamma_{k,i}\cdot X_{k,i}\cdot T_{k,i}+1/4\gamma_{k,i}\cdot T_{k,i}\cdot$$

$$(\sum_{r\varepsilon R_k} U_{r,i}-U_{r,i})+\gamma_{k,i}\cdot U_{k,i}\cdot Z_{k,i}]] \qquad (6.21)$$

The augmented cost function, $\hat{J}$, is obtained by adjoining to the cost function in (6.21) the equality constraints of Eq. (6.2) via Lagrange multipliers and the inequality constraints of Eqs. (6.3), (6.4), (6.7), and (6.9) via Kuhn-Tucker multipliers;one thus obtains

$$\hat{J}=E[\sum_{i=1}^{m}(1-\pi_i)\cdot\sum_{k=1}^{n}[b_{k,i}\cdot U_{k,i}+d_{k,i}\cdot U_{k,i}\cdot X_{k,i-1}+1/2.d_{k,i}\cdot U_{k,i}\cdot$$

$$(\sum_{r\varepsilon R_k} U_{r,i}-U_{k,i})+\gamma_{k,i}\cdot X_{k,i}\cdot T_{k,i}+1/4.\gamma_{k,i}\cdot T_{k,i}\cdot$$

$$(\sum_{r\varepsilon R_k} U_{r,i}-U_{r,i})+\gamma_{k,i}\cdot Z_{k,i}]+\sum_{k=1}^{n}[\lambda_{k,i}^{1}(-X_{k,i}+X_{k,i-1}+INF_{k,i}+\sum_{r\varepsilon R_k} U_{r,i}$$

$$-U_{k,i}+\sum_{r\in R_k} S_{r,i}-S_{k,i})+\lambda_{k,i}^{2}[-T_{k,i}+U_{k,i}.(\sum_{r\in R_k} U_{r,i}$$

$$-U_{k,i})]+\lambda_{i}^{3}.[-Z_{k,i}+X_{k,i-1}^{2}]+e_{k,i}^{1}.(\underline{U}_{k,i}-U_{k,i})$$

$$+e_{k,i}^{2}.(U_{k,i}-\overline{U}_{k,i})+e_{k,i}^{3}.(\underline{X}_{k,i}-X_{k,i})$$

$$+e_{k,i}^{4}.(X_{k,i}-\overline{X}_{k,i})+\sigma_{k,i}.[\psi_k.(X_{k,i-1}-X_{k,i})$$

$$+\xi_k.(X_{k,i-1}^{2}-X_{k,i}^{2}))]+\pi_i.[\ell_i+C]]] \qquad (6.22)$$

In the above equation, $\lambda_{k,i}^1$, $\lambda_{k,i}^2$, and $\lambda_{k,i}^3$ are Lagrange multipliers which will be determined in such a way that the corresponding equality constraints must be satisfied. $e_{k,i}^1$, $e_{k,i}^2$, $e_{k,i}^3$, $e_{k,i}^4$, $\sigma_{k,i}$, and π_i are Kuhn-Tucker multipliers. They are equal to zero if the constraints are satisfied and greater than zero if the constraints are violated.

Eq. (6.22) can be written in the following form

$$\hat{J}=E[\sum_{i=1}^{m} (1-\pi_i).[b^T(I).U(I)+d^T(I).U(I).\vec{H}.X(I-1)$$

$$+1/2.d^T(I).U(I).\vec{H}.M.U(I)+\gamma^T(I).X(I-1).\vec{H}.T(I)$$

$$+1/4.\gamma^T(I).T(I).\vec{H}.M.U(I)+\gamma^T(I).U(I).\vec{H}.Z(I)]$$

$$+\lambda_1^T(I).[-X(I)+X(I-1)+INF(I)+M.U(I)$$

$$+M.S(I)]+\lambda_2^T(I).[-T(I)+U(I).\vec{H}.M.U(I)]$$

$$+\lambda_3^T(I).[-Z(I)+X(I-1).\vec{H}.X(I-1)]+\theta_1^T(I).U(I)$$

$$+\theta_2^T(I).X(I)+\sigma^T(I).[\psi.\vec{H}.[X(I-1)-X(I)]$$

$$+\xi.\vec{H}.[X(I-1).\vec{H}.X(I-1)-X(I).\vec{H}.X(I)]]] \qquad (6.23)$$

where the following are nx1 vectors

$$b(I)=\text{col.}[b_{1,i},b_{2,i}\cdots,b_{k,i},\cdots,b_{n,i}] \qquad (6.24)$$

$$U(I)=\text{col.}[U_{1,i},U_{2,i},\cdots,U_{k,i},\cdots,U_{n,i}] \qquad (6.25)$$

$$X(I)=\text{col.}[X_{1,i},X_{2,i},\cdots,X_{k,i},\cdots,X_{n,i}] \qquad (6.26)$$

$$d(I)=\text{col.}[d_{1,i},d_{2,i},\cdots,d_{k,i},\cdots,d_{n,i}] \qquad (6.27)$$

$$\gamma(I)=\text{col.}[\gamma_{1,i},\gamma_{2,i},\cdots,\gamma_{k,i},\cdots,\gamma_{n,i}] \qquad (6.28)$$

$$Z(I)=\text{col.}[Z_{1,i},Z_{2,i},\cdots,Z_{k,i},Z_{n,i}] \qquad (6.29)$$

$$\alpha(I)=\text{col.}[\alpha_{1,i},\alpha_{2,i},\cdots,\alpha_{k,i},\cdots,\alpha_{n,i}] \qquad (6.30)$$

$$\beta(I)=\text{col.}[\beta_{1,i},\beta_{2,i},\cdots,\beta_{k,i},\cdots,\beta_{n,i}] \qquad (6.31)$$

$$\gamma(I)=\text{col.}[\gamma_{1,i},\gamma_{2,i},\cdots,\gamma_{k,i},\cdots,\gamma_{n,i}] \qquad (6.32)$$

$$\lambda_1(I)=\text{col.}[\lambda_{1,i}^{1},\lambda_{2,i}^{1},\cdots,\lambda_{k,i}^{1},\cdots,\lambda_{n,i}^{1}] \qquad (6.33)$$

$$\lambda_2(I)=\mathrm{col.}[\lambda_{1,i}^{2},\lambda_{2,i}^{2},\ldots,\lambda_{k,i}^{2},\ldots,\lambda_{n,i}^{2}] \qquad (6.34)$$

$$\lambda_3(I)]=\mathrm{col.}\{\lambda_{1,i}^{3},\lambda_{2,i}^{3},\ldots,\lambda_{k,i}^{3},\ldots,\lambda_{n,i}^{3}\} \qquad (6.35)$$

$$e_1(I)=\mathrm{col.}[e_{1,i}^{1},e_{2,i}^{1},\ldots,e_{k,i}^{1},\ldots,e_{n,i}^{1}] \qquad (6.36)$$

$$e_2(I)=\mathrm{col.}[e_{1,i}^{2},e_{2,i}^{2},\ldots,e_{k,i}^{2},\ldots,e_{n,i}^{2}] \qquad (6.37)$$

$$e_3(I)=\mathrm{col.}[e_{1,i}^{3},e_{2,i}^{3},\ldots,e_{k,i}^{3},\ldots,e_{n,i}^{3}] \qquad (6.38)$$

$$e_4(I)=\mathrm{col.}[e_{1,i}^{4},e_{2,i}^{4},\ldots,e_{k,i}^{4},\ldots,e_{n,i}^{4}] \qquad (6.39)$$

$$\theta_1(I)=\mathrm{col.}[\theta_{1,i}^{1},\theta_{2,i}^{1},\ldots,\theta_{k,i}^{1},\ldots,\theta_{n,i}^{1}] \qquad (6.40)$$

$$\theta_2(I)=\mathrm{col.}[\theta_{1,i}^{2},\theta_{2,i}^{2},\ldots,\theta_{k,i}^{2},\ldots,\theta_{n,i}^{2}] \qquad (6.41)$$

$$T(I)=\mathrm{col.}[T_{1,i},T_{2,i},\ldots,T_{k,i},\ldots,T_{n,i}] \qquad (6.42)$$

$$\sigma(I)=\mathrm{col.}[\sigma_{1,i},\sigma_{2,i},\ldots,\sigma_{k,i},\ldots,\sigma_{n,i}] \qquad (6.43)$$

$$\psi=\mathrm{col.}[\psi_1,\psi_2,\ldots,\psi_k,\ldots,\psi_n] \qquad (6.44)$$

$$\xi=\mathrm{col.}[\xi_1,\xi_2,\ldots,\xi_k,\ldots,\xi_n] \qquad (6.45)$$

$$INF(I)=\mathrm{col.}[INF_{1,i},INF_{2,i},\ldots,INF_{k,i},\ldots,INF_{n,i}] \qquad (6.46)$$

$$S(I)=\mathrm{col.}[S_{1,i},S_{2,i},\ldots,S_{k,i},\ldots,S_{n,i}] \qquad (6.47)$$

and M is an nxn matrix where the diagonal elements are equal to (-1), and the other elements vary between (one) and (zero) depending on the topological arrangement of the reservoirs as explained in the example given in Chapter IV. $\vec{H}$ is a vector matrix in which the vector index varies from 1 to n, while the matrix dimension of $\vec{H}$ is nxn, [3], as explained in Chapter V. Moreover, the values of $\theta_{k,i}^{1}$ and $\theta_{k,i}^{2}$ are defined as

$$\theta_{k,i}^{1} = e_{k,i}^{1} - e_{k,i}^{2} \tag{6.48}$$

$$\theta_{k,i}^{2} = e_{k,i}^{3} - e_{k,i}^{4} \tag{6.49}$$

Constant terms are dropped from Eq. (6.23). The values of

$$\sum_{i=1}^{m} \lambda_1^{T}(I).X(I) = \sum_{i=1}^{m} \lambda_1^{T}(I-1).X(I-1) + \lambda_1^{T}(m).X(m) - \lambda_1^{T}(0).X(0) \tag{6.50}$$

$$\sum_{i=1}^{m} \theta_2^{T}(I).X(I) = \sum_{i=1}^{m} \theta_2^{T}(I-1).X(I-1) + \theta_2^{T}(m).X(m) - \theta_2^{T}(0).X(0) \tag{6,51}$$

$$\sum_{i=1}^{m} \sigma^{T}(I).\psi.\vec{H}.X(I) = \sum_{i=1}^{m} \sigma^{T}(I-1).\psi.\vec{H}.X(I-1) + \sigma^{T}(m).\psi.\vec{H}.X(m) - \sigma^{T}(0).\psi.\vec{H}.X(0) \tag{6,52}$$

and

$$\sum_{i=1}^{m} \sigma^T(I).\xi.\vec{H}.X(I).\vec{H}.X(I)$$

$$= \sigma^T(I-1).\xi.\vec{H}.X(I-1).\vec{H}.X(I-1)$$

$$+ \sigma^T(m).\xi.\vec{H}.X(m).\vec{H}.X(m)$$

$$- \sigma^T(0).\xi.\vec{H}.X(0).\vec{H}.X(0) \qquad (6.53)$$

Also, defining the following vectors

$$W^T(I) = [X^T(I-1) \quad U^T(I) \quad T^T(I) \quad Z^T(I)] \qquad (6.54)$$

$$R(I) = \begin{bmatrix} \lambda_1(I)-\lambda_1(I-1)+\theta_2(I-1)-\psi^T.\vec{H}.[\sigma(I)-\sigma(I-1)] \\ (1-\pi_1).b(I)+\theta_1(I)+M^T.\lambda_1(I) \\ -\lambda_2(I) \\ -\lambda_3(I) \end{bmatrix} \qquad (6.55)$$

$$L(I)=\begin{bmatrix} \begin{array}{l}\lambda_3(I).\vec{H}\\ +\xi.\vec{H}.[\sigma(I)-\sigma(I-1)].\vec{H}\end{array} & \frac{1}{2}(1-\pi_i).d(I).\vec{H} & \frac{1}{2}(1-\pi_i).\gamma(I).\vec{H} & 0 \\ \frac{1}{2}(1-\pi_i).d(I).\vec{H} & \begin{array}{l}\frac{1}{4}(1-\pi_i).d^T(I).\vec{H}.M\\ +\frac{1}{4}(1-\pi_i).M^T.\vec{H}.d(I)\\ +\frac{1}{2}.\lambda_2{}^T(I).\vec{H}.M\\ +\frac{1}{2}.M^T.\vec{H}.\lambda_2(I)\end{array} & \frac{1}{8}(1-\pi_i).M^T.\vec{H}.\gamma(I) & \frac{1}{2}(1-\pi_i).\gamma(I).\vec{H} \\ \frac{1}{2}(1-\pi_i).\gamma(I).\vec{H} & \frac{1}{8}(1-\pi_i).\gamma^T(I).\vec{H}.M & 0 & 0 \\ 0 & \frac{1}{2}(1-\pi_i).\gamma(I).\vec{H} & 0 & 0 \end{bmatrix} \qquad (6.56)$$

we obtain the augmented cost function

$$\hat{J}=-\lambda^{T}(m).\mathbf{X}(m)+\theta_{2}^{T}(m).\mathbf{X}(m)-\sigma^{T}(m).\psi.\vec{\mathbf{H}}.\mathbf{X}(m)$$

$$-\ \sigma^{T}(m).\xi.\vec{\mathbf{H}}.\mathbf{X}(m).\vec{\mathbf{H}}.\mathbf{X}(m)$$

$$+\ \sum_{I=1}^{m}\ \{\mathbf{W}^{T}(I).\mathbf{L}(I).\mathbf{W}(I)+\mathbf{R}^{T}(I).W(I)\} \qquad (6.57)$$

constant parts have been dropped from equation (6.57).

Equation (6.57) is composed of a boundary part and a discrete integral part; these are independent of each other so, Eq. (6.57) can be written as

$$\hat{J} = \hat{J}_1 + \hat{J}_2 \qquad (6.58)$$

where

$$\hat{J}_1=-\lambda_1^{T}(m).\mathbf{X}(m)+\theta_{2}^{T}(m).\mathbf{X}(m)-\sigma^{T}(m).\psi.\vec{\mathbf{H}}.\mathbf{X}(m)$$

$$-\ \sigma^{T}(m).\xi.\vec{\mathbf{H}}.\mathbf{X}(m).\vec{\mathbf{H}}.\mathbf{X}(m) \qquad (6.59)$$

and

$$\hat{J}_2= \sum_{I=1}^{m} [\mathbf{W}^{T}(I).\mathbf{L}(I).\mathbf{W}(I)+\mathbf{R}^{T}(I).\mathbf{W}(I)] \qquad (6.60)$$

If we define the vector V(I) such that

$$\mathbf{V}(I)=\mathbf{L}^{-1}(I).\mathbf{R}(I) \qquad (6.61)$$

then, the augmented cost function in Eq. (6.60) can be written as

$$\hat{J}_2=\sum_{I=1}^{m}[[\mathbf{W}(I)+1/2\mathbf{V}(I)]^T.\mathbf{L}(I).[\mathbf{W}(I)+1/2\mathbf{V}(I)]$$

$$-1/4\mathbf{V}(I).\mathbf{L}^T(I).\mathbf{V}(I)] \qquad (6.62)$$

$\mathbf{V}(I)$ and $\mathbf{L}(I)$ are independent of $\mathbf{W}(I)$ then, dropping the constant terms from Eq. (6.59) we obtain

$$\hat{J}_2=\sum_{I=1}^{m}[[\mathbf{W}(I)+1/2\mathbf{V}(I)]^T.\mathbf{L}(I).[\mathbf{W}(I)+1/2\mathbf{V}(I)]] \qquad (6.63)$$

Equation (6.63) defines a norm in Hilbert space; hence, we can rewrite it as

$$\hat{J}_2=||\mathbf{W}(I)+1/2\mathbf{V}(I)||_{\mathbf{L}(I)} \qquad (6.64)$$

6.6 The Optimal Solution

To maximize $\hat{J}$ in Eq. (6.58) we will maximize each term separately, i.e.,

$$\underset{[\mathbf{X}(m),\mathbf{W}(I)]}{\mathrm{Max}.\hat{J}}=\underset{[\mathbf{X}(m)]}{\mathrm{Max}.\hat{J}_1}+\underset{[\mathbf{W}(I)]}{\mathrm{Max}.\hat{J}_2} \qquad (6.65)$$

The maximum of $\hat{J}_1$ is achieved when

$$-\lambda_1(m)+\theta_2(m)-\psi^T.\mathbf{H}.\sigma(m)-2\xi^T.\vec{\mathbf{H}}.\sigma(m).\vec{\mathbf{H}}.\mathbf{X}.(m)=\underline{0} \qquad (6.66)$$

The maximum of $\hat{J}_2$ of Eq. (6.64) is achieved when

$$\mathbf{W}(I)+1/2\mathbf{V}(I)=\underline{0} \qquad (6.67)$$

substituting from Eq. (6.58) into Eq. (6.64), we obtain

$$\mathbf{R}(I)+2.\mathbf{L}(I).\mathbf{W}(I)=\underline{0} \qquad (6.68)$$

Writing the above equation explicitly using Eqs. (6.54), (6.55), and (6.56) and adding the equality constraints in Eqs. (6.2), (6.17), (6.18), (6.19), (6.20), (6.48), and (6.49), we get

$$\begin{aligned}&\lambda_1(I)-\lambda_1(I-1)+\theta_2(I-1)-\psi^T.\vec{\mathbf{H}}.[\sigma(I)-\sigma(I-1)]\\ &\quad+2.\lambda_3(I).\vec{\mathbf{H}}.\mathbf{X}(I-1)+\xi.\vec{\mathbf{H}}.[\sigma(I)-\sigma(I-1)].\vec{\mathbf{H}}.\mathbf{X}(I-1)\\ &\quad+[1-\pi(I)].\mathbf{d}(I).\vec{\mathbf{H}}.\mathbf{U}(I)+[1-\pi(I)].\gamma(I).\vec{\mathbf{H}}.\mathbf{T}(I)=\underline{0} \qquad (6.69)\end{aligned}$$

$$\begin{aligned}&[1-\pi(I)].\mathbf{b}(I)+\theta_1(I)+\mathbf{M}^T.\lambda_1(I)+[1-\pi(I)].\mathbf{d}(I).\vec{\mathbf{H}}.\mathbf{X}(I-1)\\ &\quad+1/2[1-\pi(I)].\mathbf{d}^T(I).\mathbf{H}.\mathbf{M}.\mathbf{U}(I)+1/2[1-\pi(I)].\mathbf{M}^T.\vec{\mathbf{H}}.\mathbf{d}(I).\mathbf{U}(I)\\ &\quad+\lambda_2^T(I).\vec{\mathbf{H}}.\mathbf{M}.\mathbf{U}(I)+\mathbf{M}^T.\vec{\mathbf{H}}.\lambda_2(I).\mathbf{U}(I)\\ &\quad+1/4[1-\pi(I)].\mathbf{M}^T.\vec{\mathbf{H}}.\gamma(I).\mathbf{T}(I)+[1-\pi(I)].\gamma(I).\vec{\mathbf{H}}.\mathbf{Z}(I)=\underline{0}\end{aligned} \qquad (6.70)$$

$$-\lambda_2(I)+[1-\pi(I)].\gamma(I).\vec{\mathbf{H}}.\mathbf{X}(I-1)$$

$$+1/4[1-\pi(I)].\gamma^T(I).\vec{\mathbf{H}}.\mathbf{M}.\mathbf{U}(I) = \underline{0} \qquad (6.71)$$

$$-\lambda_3(I)+[1-\pi(I)].\gamma(I).\vec{\mathbf{H}}.\mathbf{U}(I) = \underline{0} \qquad (6.72)$$

$$-\mathbf{X}(I)+\mathbf{X}(I-1)+\mathbf{INF}(I)+\mathbf{M}.\mathbf{U}(I)+\mathbf{M}.\mathbf{S}(I)=\underline{0} \qquad (6.73)$$

$$-\mathbf{b}(I)+\alpha(I)+1/2.\beta(I).\vec{\mathbf{H}}.[\mathbf{INF}(I)+\mathbf{M}.\mathbf{S}(I)]$$

$$+1/4.\gamma(I).\vec{\mathbf{H}}.[\mathbf{INF}(I).\vec{\mathbf{H}}.\mathbf{INF}(I)+\mathbf{S}^T(K).\mathbf{M}^T(I).\vec{\mathbf{H}}.\mathbf{M}.\mathbf{S}(I)$$

$$+2.\mathbf{INF}(I).\vec{\mathbf{H}}.\mathbf{M}.\mathbf{S}(I)] = \underline{0} \qquad (6.74)$$

$$-\mathbf{d}(I)+\beta(I)+\gamma(I).\vec{\mathbf{H}}.[\mathbf{INF}(I)+\mathbf{M}.\mathbf{S}(I)]=\underline{0} \qquad (6.75)$$

$$-\mathbf{Z}(I)+\mathbf{X}(I-1).\vec{\mathbf{H}}.\mathbf{X}(I-1) = \underline{0} \qquad (6.76)$$

$$-\mathbf{T}(I)+\mathbf{U}(I).\vec{\mathbf{H}}.\mathbf{M}.\mathbf{U}(I) = \underline{0} \qquad (6.77)$$

$$-\theta_1(I)+\mathbf{e}_1(I)-\mathbf{e}_2(I) = \underline{0} \qquad (6.78)$$

$$-\theta_2(I)+\mathbf{e}_3(I)-\mathbf{e}_4(I) = \underline{0} \qquad (6.79)$$

We also have the following limits on Kuhn-Tucker values

$$e_{k,i}^{1} = \begin{cases} 0.0 & \text{if } U_{k,i} \geq \underline{U}_{k,i} \\ >0.0 & \text{if } U_{k,i} < \underline{U}_{k,i} \end{cases} \qquad (6.80)$$

$$e_{k,i}^{2} = \begin{cases} 0.0 & \text{if } U_{k,i} \leq \overline{U}_{k,i} \\ >0.0 & \text{if } U_{k,i} > \overline{U}_{k,i} \end{cases} \qquad (6.81)$$

$$e_{k,i}^{3} = \begin{cases} 0.0 & \text{if } X_{k,i} \geq \underline{X}_{k,i} \\ >0.0 & \text{if } X_{k,i} < \underline{X}_{k,i} \end{cases} \qquad (6.82)$$

$$e_{k,i}^{4} = \begin{cases} 0.0 & \text{if } X_{k,i} \leq \overline{X}_{k,i} \\ >0.0 & \text{if } X_{k,i} > \overline{X}_{k,i} \end{cases} \qquad (6.83)$$

$$\sigma_{k,i} = \begin{cases} 0.0 & \text{if } \psi_k \cdot (X_{k,i-1} - X_{k,i}) + \xi_k \cdot (X_{k,i-1}^{2} - X_{k,i}^{2}) \leq \overline{D}_k \\ >0.0 & \text{if } \psi_k \cdot (X_{k,i-1} - X_{k,i}) + \xi_k \cdot (X_{k,i-1}^{2} - X_{k,i}^{2}) > \overline{D}_k \end{cases}$$

(6.84)

$$\pi_i = \begin{cases} 0.0 & \text{if} \quad \sum_{k=1}^{n} G_{k,i} - \ell_i \geq C \\ >0.0 & \quad\;\; \sum_{k=1}^{n} G_{k,i} - \ell_i < C \end{cases} \qquad (6.85)$$

Equations (6.69) to (6.85) with Equation (6.66) completely specify optimal solution.

6.7 Algorithm for Solution

Given a system of n reservoirs connected like a tree (this can fit any arbitrary topological arrangement), the expected values of natural inflows, $INF_{k,i}$, the initial storage, $X_{k,o}$, usually all reservoirs are full at the beginning of the critical period ($X_{k,o}=\overline{X}_{k,o}$) unless drafting for minimum flow or flood control, and the tables which specify the relationships between the variables at each plant in the system and different types of equality and inequality constraints:

1) assume initial value for $X_{k,i}$, $k=1,..,n;\ i=1,...,m$

2) check the limits on $X_{k,i}$, so that

   if $X_{k,i}<\underline{X}_{k,i}$ let $X_{k,i}=\underline{X}_{k,i}$

   if $X_{k,i}>\overline{X}_{k,i}$ let $X_{k,i}=\overline{X}_{k,i}$

3) assume the spill at the beginning is equal to zero, i.e.

   $S_{k,i}=0.0 \quad k=1,...,n;\ i=1,...,m$

4) calculate the outflow $U_{k,i}$ using the water conservation equation

$$U_{k,i}=-X_{k,i}+X_{k,i-1}+INF_{k,i}+\sum_{r\varepsilon R_k}(U_{r,i}+S_{r,i})-S_{k,i}$$

   To do so, start at the beginning of any branch. Then, go to the next down-stream plant till the end of that branch. Then, go to the next branch and so on.

5) check the limits on $U_{k,i}$, so that

if $U_{k,i} < \underline{U}_{k,i}$ let i) $u_{k,i} = \underline{U}_{k,i}$, and

ii) calculate the corresponding value of $X_{k,i}$ using the water conservation equation.

if $U_{k,i} > \overline{U}_{k,i}$ let i) $U_{k,i} = \overline{U}_{k,i}$, and

ii) calculate the corresponding value of $X_{k,i}$ using the water conservation equation

6) check the limits on $X_{k,i}$

if $X_{k,i} < \underline{X}_{k,i}$ let i) $X_{k,i} = \underline{X}_{k,i}$,

ii) calculate the corresponding value of $U_{k,i}$ using the water conservation equation, and

iii) $e_{k,i}^{1} > 0.0$

if $X_{k,i} > \overline{X}_{k,i}$ let i) $X_{k,i} = \overline{X}_{k,i}$

ii) calculate the corresponding value of $S_{k,i}$ using the water conservation equation, and

iii) $e_{k,i}^{2} > 0.0$

if $\psi_k \cdot (X_{k,i-1} - X_{k,i}) + \xi_k \cdot (X_{k,i-1}^{2} - X_{k,i}^{2}) > \overline{D}_k$

let $\sigma_{k,i} > 0.0$

if $X_{k,i}$ and/or $U_{k,i}$ satisfy the boundary constraints given in equations (6.80) to (6.84), then the corresponding values of Kuhn-Tucker multipliers will be set equal to zero.

7) calculate the corresponding value of the tailwater elevation $TWE_{k,i}$ using the field measured table of the discharge versus tailwater elevation ($U_k + S_k$ vs TWE_k) using a cubic spline subroutine, Appendix 1.

8) construct a new table between the storage and the head (X_k vs $H_{k,i}$) using the field measured table of the storage versus forebay

elevation where $H_{k,i}=E_k-TWE_{k,i}$.

9) match the (X_k vs $H_{k,i}$) table with the field measured table of the water conversion factor versus the effective head, (WCF_k vs H_k) using a suitable cubic spline subroutine to conclude with a new table of the storage versus the water conversion factor $(X_k \text{ vs } WCF_k)_i$

10) calculate $\alpha_{k,i}$, $\beta_{k,i}$, and $\gamma_{k,i}$ for the $(X_k \text{ vs } WCF_k)_i$ table using a least squares curve fitting subroutine

11) calculate the total energy generation $G_{k,i}$ $k=1,..,n$; $i=1,...,m$, where

$$G_{k,i}=U_{k,i}\cdot[\alpha_{k,i}+1/2\beta_{k,i}(X_{k,i}+X_{k,i-1})+1/4\gamma_{k,i}(X_{k,i}+X_{k,i-1})]$$

12) calculate the value of $\theta_{k,i}^{1}$ and $\theta_{k,i}^{2}$ using Eqs. (6.78) and (6.79) given the values of ψ_k and ξ_k calculate the value of $\lambda_{k,m}^{1}$, $k=1,...,n$ using Eq. (6.66)

13) calculate the values of π_i $i=1,...,m$ using Eq. (6.85)

14) calculate $\lambda_{k,i}^{2}$ $k=1,...,n$; $i=1,...,m$, using Eq. (6.71)

15) calculate $\lambda_{k,i}^{3}$ $k=1,...,n$; $i=1,...,m$, using Eq. (6.72)

16) calculate the value of $Z_{k,i}$ $k=1,...,n$; $i=1,...,m$ using Eq. (6.76)

18) calculate the value of $T_{k,i}$ $k=1,...,n$; $i=1,...,m$, using Eq. (6.77)

19) calculate the value of $b_{k,i}$ $k=1,...,n$; $i=1,...,m$, using Eq. (6.74)

20) calculate $d_{k,i}$ $k=1,...,n$; $i=1,...,m$ using Eq. (6.75)

21) calculate $\lambda_{k,i}^{1}$ $k=1,...,n$ and $i=m-1$, $m-2,...,1$ (backward) given the value of $\lambda_{k,m}^{1}$, step 8, using Eq. (6.69)

22) calculate the new value of $X_{k,i}$

$$X(I-1)=[[1-\pi(I)].d(I).\vec{H}]^{-1}.[[1-\pi(I)].[b(I)+d(I).\vec{H}.X(I-1)$$

$$+1/2.d^{T}(I).\vec{H}.M.U(I)+1/2M^{T}.\vec{H}.d(I).U(I)+1/4.M^{T}.\vec{H}.\gamma(I).T(I)$$

$$+\gamma(I).\vec{H}.Z(I)]+\theta_{1}(I)+M^{T}.\lambda_{1}(I)+\lambda_{2}^{T}(I).\vec{H}.M.U(I)$$

$$+\ M^{T}.\vec{H}.\lambda_{2}(I).U(I)]$$

```
23) check the boundary conditions on $X_{k,i}$, so that
    if $X_{k,i}>\overline{X}_{k,i}$ let $X_{k,i}=\overline{X}_{k,i}$
    if $X_{k,i}<\underline{X}_{k,i}$ let $X_{k,i}=\underline{X}_{k,i}$
24) let $S_{k,i}=0.0$  $k=1,...,n;\ i=1,...,m.$
25) calculate $U_{k,i}$ using the water conservation equation
26) check the limits on $U_{k,i}$, so that
    if $U_{k,i}<\underline{U}_{k,i}$  i)  let $U_{k,i}=\underline{U}_{k,i}$, and
                      ii) calculate the corresponding value of $X_{k,i}$
                          using the water conservation equation
    if $U_{k,i}>\overline{U}_{k,i}$  i)  let $U_{k,i}=\overline{U}_{k,i}$, and
                      ii) calculate the corresponding value of $X_{k,i}$
                          using the water conservation equation
27) check the limits on $X_{k,i}$, so that
    if $X_{k,i}<\underline{X}_{k,i}$  i)  let $X_{k,i}=\underline{X}_{k,i}$
                       ii) calculate the corresponding value of $U_{k,i}$
                           using the water conservation equation, and
                       iii)let $e_{k,i}^{1}>0.0$
    if $X_{k,i}>\overline{X}_{k,i}$  i)  let $X_{k,i}=\overline{X}_{k,i}$,
                       ii) calculate the corresponding value of $U_{k,i}$
                           using the water conservation equation
                       iii)let $e_{k,i}^{2}>0.0$
```

if $\psi_k \cdot (X_{k,i-1} - X_{k,i}) + \xi_k \cdot (X_{k,i-1}^2 - X_{k,i}^2) > \bar{D}_k$ let $\sigma_{k,i} > 0.0$

28) calculate the total generation $G_{k,i}$, $k=1,\ldots,n$; $i=1,\ldots,m$ after calculating the values of $\alpha_{k,i}$, $\beta_{k,i}$ and $\gamma_{k,i}$ (steps 7 to 11)

29) calculate the value of the objective function, J, where

$$J = \sum_{i=1}^{m} \sum_{k=1}^{n} G_{k,i}$$

30) if the solution converges toward a better solution, then continue (go to step 12) until no significant changes occur from iteration to iteration.

if the solultion diverges then

i) stop the calculation

ii) use the results obtained to modify the controlling soft constraints $\hat{U}_{k,i}$ and $\hat{X}_{k,i}$, and

iii) repeat the calculation (start at step 1).

6.8 Application to the B.P.A. Hydro System

The problem of the load following optimization of the B.P.A. hydro system is carried on considering all the requirements proposed in this chapter. As we can see in Figs. 13 and 14 the essential need for a load following optimization technique. Fig. 13 compares the dependable load and the corresponding hydro generation obtained from the given rule curves used in the B.P.A. hydro corporation. By "the dependable load" we mean the value of load obtained after subtracting from the total firm load an amount representing the required thermal, nuclear, solar, wind, wave and other sources of power generation. The fulfilment of this dependable load means no curtailment of firm load is

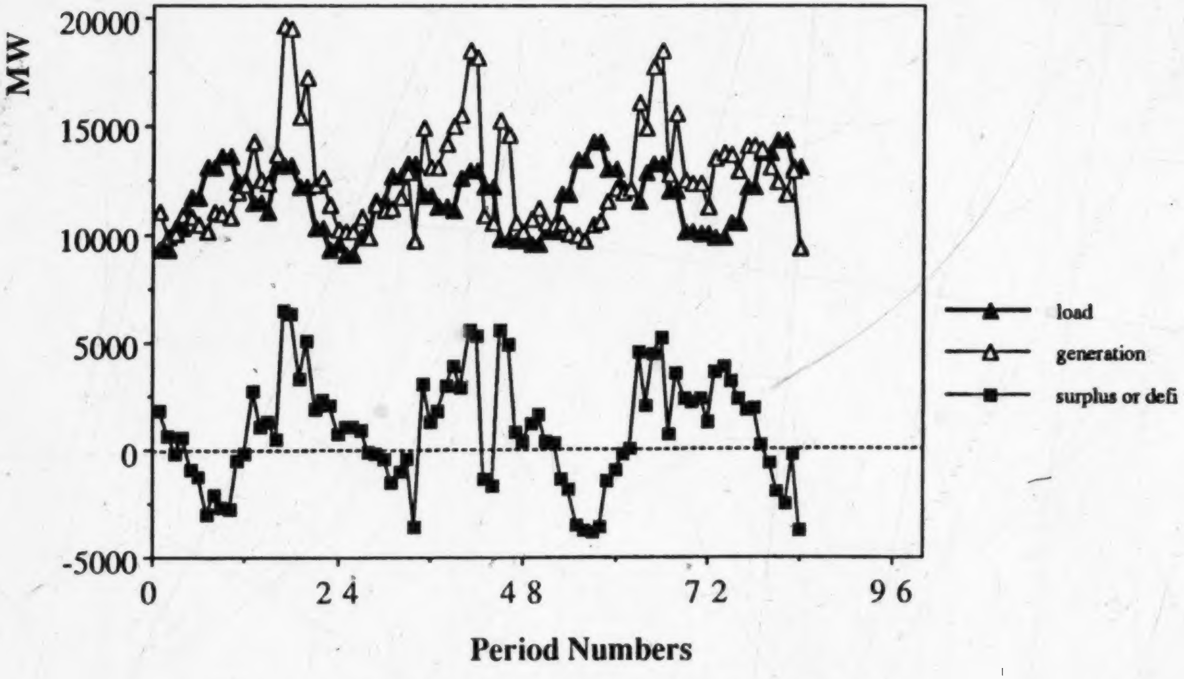

Fig. 13 B.P.A. Rule Curves Results.

necessary and no need to import an expensive energy to meet the deficit. In Fig. 15 the cost or profit arising from W_i is represented by the function C_i called the energy deficit generation cost. Fig. 14 presents the resulting total generation versus the load using the results obtained from Chapter V during each period. And Fig. 16 presents the same comparison for the load following algorithm presented in this chapter. A comparison between Figs. 13, 14, and 16 proves the capability of this algorithm to maximize the total energy generated while it guarantees the most uniform surplus power.

In Fig. 16 there is some nonuniformity which cannot be avoided due to the imposed constraints on the scheduling process. One of the major causes of the nonuniformity is the water budget requirement (Chapter III). For example, the required minimum flow at Grand Coulee and Priest Rapids Dams jumps from 50,000 or 60,000 CFS to 134,000 CFS during May to satisfy the Water Budget minimum flow on the Columbia River. This makes the total generation during May peak more than during any other period and cause this unavoidable nonuniformity.

In the programming process, we have compiled six projects in pairs to decrease the computational effort as discussed in Chapter IV. Also, a special subroutine is made to suit the special configuration of the Canal Plant Dam, Fig. 9, and the required distribution of the water budget between the Canal Plant and Corra Linn Dams, Table 126. Furthermore, a special subroutine is made to calculate the corresponding tailwater for a given release to Yale and Upper Baker Dams because of the sharp changes occurring in their field measured tables, Fig. 12.

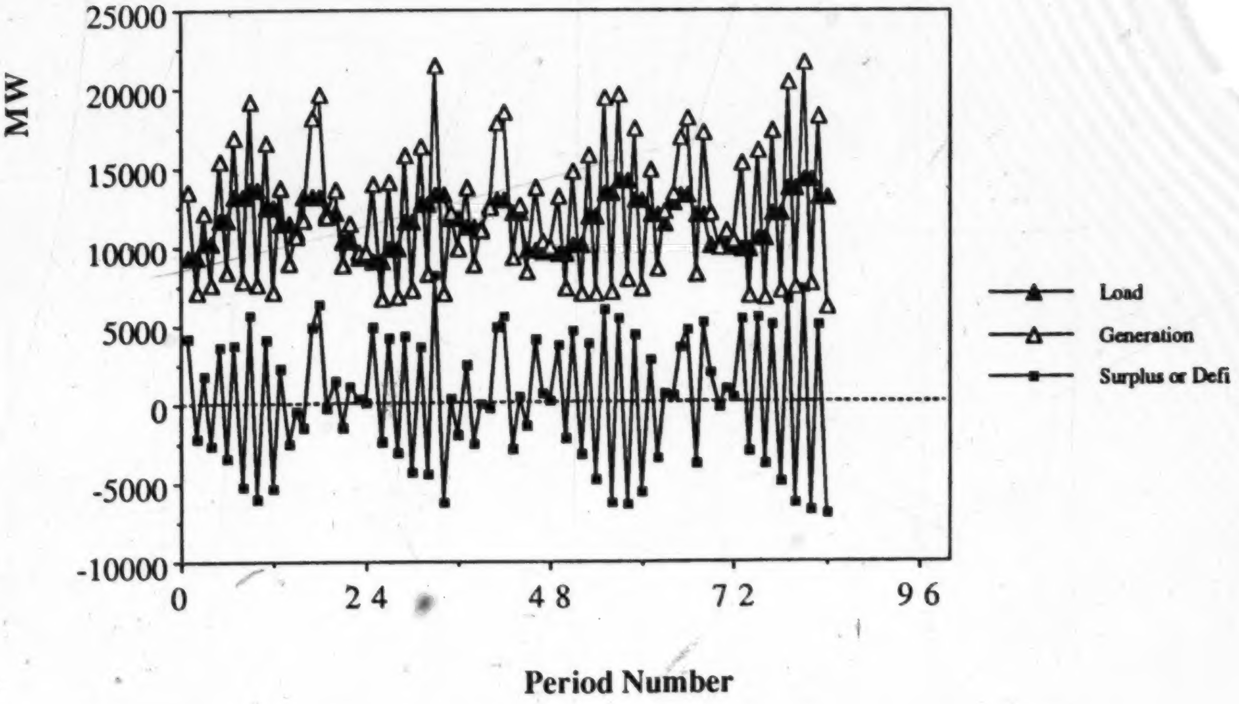

Fig. 14 Chapter 5 Rule Curves Results .

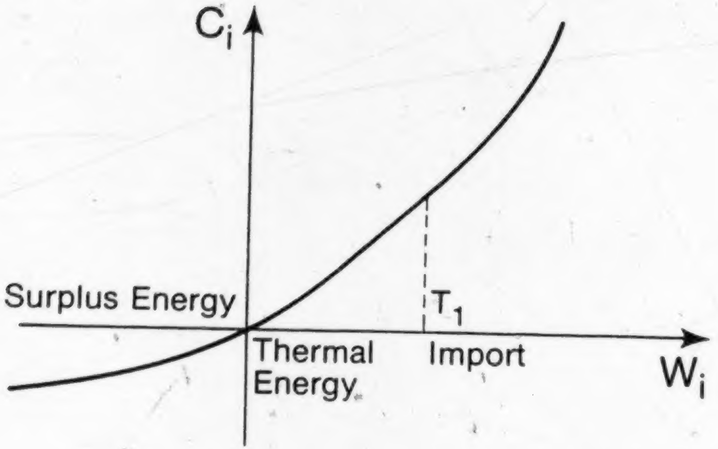

Fig. 15 Cost Function of the Energy Deficit in Period i.

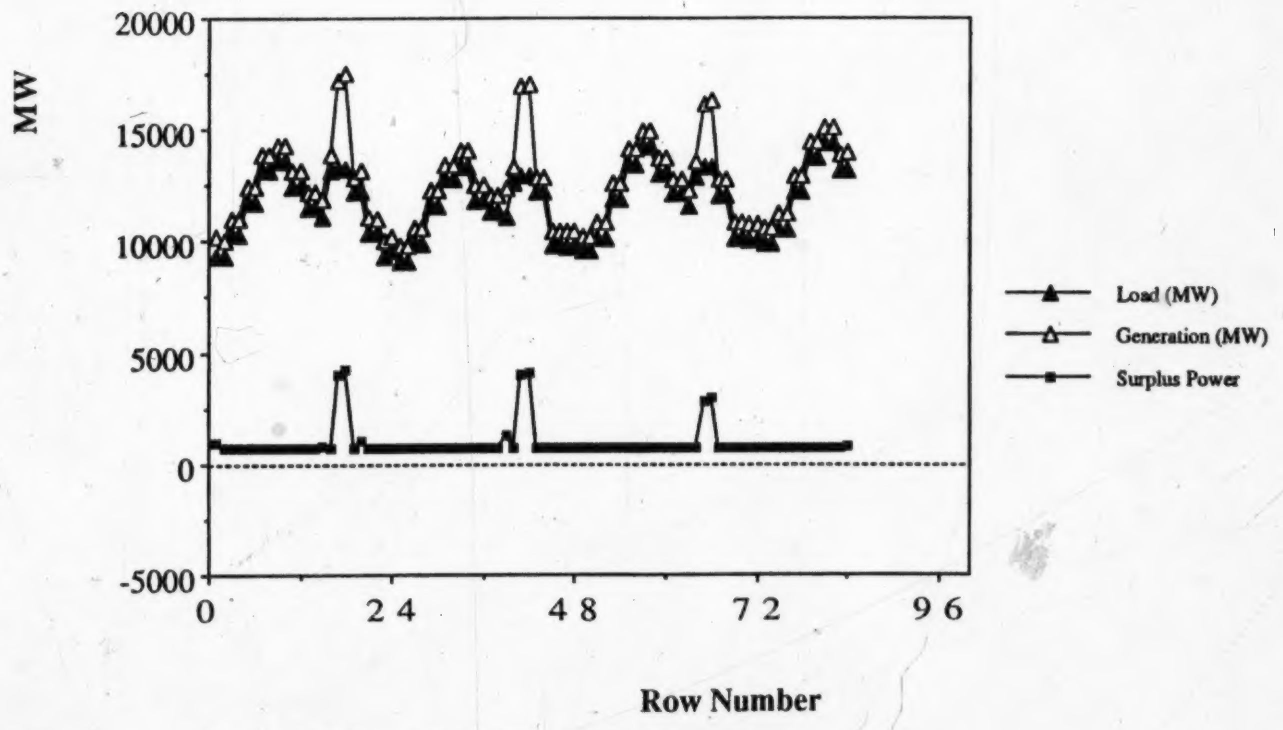

Fig. 16 Load Following Results .

In Table 15, we compare the amount of the uniform surplus power obtained by B.P.A. rule curves and that obtained in Chapter V to that obtained using the load following algorithm in this chapter. In Table 15 a comparison of the average power generated by the B.P.A. rule curves results, the calculated results in Chapter V, and that obtained by the load following technique is presented. In Table 17 a comparison of the total energy capability for the B.P.A. rule curves results, the results of Chapter V and the load following results.

Table 15

A comparison of the violation of the minimum discharge requirement, the spill, and the amount of the uniform surplus power for the B.P.A. rule curves results, the results of Chapter V, and that using the load following technique.

	$U_{k,i}<\underline{U}_{k,i}$	Spillage	Uniform Surplus Power
B.P.A. rule curve results	this happens 68 times during the 84 critical period for the 88 project $\sum \underline{U}_{k,i}-U_{k,i}=48,845.8$ $\underline{U}_{k,i}>U_{k,i}$	$\sum_{k,i} S_{k,i}=4.378E6$	-3,828.2MW (deficit)
Chapter V results	$\sum \underline{U}_{k,i}-U_{k,i}=0.0$ $\underline{U}_{k,i}>U_{k,i}$	$\sum_{k,i} S_{k,i}=1.773E6$	-6,903.7MW (deficit)
Load following results	$\sum \underline{U}_{k,i}-U_{k,i}=0.0$ $\underline{U}_{k,i}>U_{k,i}$	$\sum_{k,i} S_{k,i}=1.774E6$	709.4MW (surplus)

Table 16

A comparison of the average power generated between the B.P.A. rule curve results, the calculated results of Chapter V, and the load following results.

Period		Average Power Generated by		
from	to	B.P.A.	Chapter V	load following
Sept. 1,1928	June 31,1929	12.373	12.594	13.044
July 1,1929	June 31,1930	11.801	12.511	12.377
July 1,1930	June 31,1931	12.321	12.211	12.669
July 1,1931	Feb. 29,1932	12.163	13.974	12.464

Table 17

A comparison of the total energy capability for the B.P.A. rule curve results, the results of Chapter V, and the load following results.

Comparison of	B.P.A.	Chapter V	Load following
Total Energy (GWH)	372,512	389,981	387,258
Average power (GW)	12.155	12.725	12.636
% increase over the B.P.A.		4.689%	3.967%

6.9 Conclusions

I have presented a very general framework for the formulation and solution of the hydro system scheduling problem. The framework allows for the incorporation of virtually all types of constraints that are imposed in the planning and the actual operation of hydro systems. The algorithm described in this chapter proves its ability to produce the maximum generation with the most uniform surplus power. The algorithm accurately models many of the hydro-plant parameters and constraints and optimizes the schedules by means of functional analysis and the minimum norm formulation technique.

In this chapter, the generation from each reservoir is considered as a quadratic function of the storage times the discharge through the turbine. Tailwater elevation is taken as a nonlinear function of the total release. Forebay elevation is considered as a nonlinear function of the storage.

Also in this chapter, the development and major characteristics of the computationally efficient solution methodology are described and some of the pitfalls in applying such a technique to a relatively large system are presented. Typical numerical results for an actual hydro system illustrate the capabilities of the proposed solution scheme.

The basic feature of this new procedure is its ability to produce the maximum uniform surplus, which satisfies the system constraints. The procedure can compute the system parameters for hundreds of sets of rule curves of 51 run-of-river plants and 37 reservoirs and 96 time periods. Furthermore, from the results obtained, it is evident that this technique shows much promise.

CHAPTER VII

CONCLUDING REMARKS

7.1 Conclusions

In this thesis, a presentation has been made to illustrate the solution for the half monthly operating policy of a multireservoir tree connected hydro power system. The proposed solution has been done by one-at-a-time method starting from the end reservoir or run-of-river plant of each branch and taking one branch at a time.

The problem is formulated as a minimum norm problem in the framework of functional analysis and the equations obtained are nonlinear discrete time equations. The time period used in the long range modeling is half a month; therefore, short range hydraulic and electrotechnique effects are not taken into consideration.

The tree system is the general case of Dams' topology which adequately specifies a real system and fits any arbitrary configuration. It is an improvement over the methods which deal only with independent rivers,parallel connections,with reservoirs connected in series.

The technique presents a very general framework for the formulation and solution of the hydro system scheduling problem. This framework allows for the accommodation of all types of constraints imposed in the planning and due to the actual operation of the hydro systems. The algorithm described proves its ability to produce the maximum generation with the most uniform surplus power.

Also, we present some of the pitfalls in applying such a technique to a relatively large system. The application of the technique is carried out for two systems; a nine reservoir system and the B.P.A. hydro system. The problem of scheduling the operation of the B.P.A. hydro system is one of the largest problems of its kind that has been solved to date. It involves approximately 271,462 variables, a nonlinear objective function, 8,437 linear equations, 41,760 linear and nonlinear type of inequality constraints, many auxiliary relationships and 968 tables. Because of the large problem size, special precautions had to be taken to insure convergence of the computational technique, computational accuracy, and efficiency. Considerable data handling problems also had to be overcome. The final results depend upon the accurate representation of the reservoir, run of river, and the hydro unit models coupled with the constant, variable tailwater elevation, linear quadratic water conversion factor as a function of the storage, or linear or nonlinear forebay elevation as a function of the storage.

The scheduling problem is fully specified by subjecting storages, penstock releases, spillages, and total releases to a set of linear and nonlinear constraints. Constraints play two important roles in this study: (1) they enforce feasibility due to physical and/or technical features in the system, and (2) guarantee that functions other than power generation are adequately fulfilled, by introducing suitable constraints on penstock releases, spillages, and storages so as to satisfy contractual agreements and regulations related to flood control, wildlife, irrigation, navigation, fisheries, and water quality requirements, etc.

In this thesis we solve a highly nonlinear problem considering all the effects of :

1) Reservoirs on dependent rivers,
2) Multi-valleys,
3) All linear and nonlinear, equality and inequality constraints,
4) Tailwater elevation depends on the total release,
5) Draft constraints,
6) The generation is a nonlinear function of the outflow through the turbine and the effective head. The latter is a nonlinear function of the storage,
7) Large problem with 88 projects and 96 time period.

This new technique is independent of the starting points and able to combine methodology and experience.

The basic feature of this new procedure is its ability to produce the maximum uniform power surplus, while satisfying the system constraints. It is evident from the results obtained that this technique shows much promise.

7.2 Suggestions for Further Research

The minimum norm formulation employed in this investigation has demonstrated the capability of solving complex power system scheduling problems. For example, the long-term optimal scheduling for realistic systems having thermal, hydro, and nuclear power plants as a variable source may be possible to solve using the same technique.

It may be possible to solve the short-term optimal operation for the hydro and hydro-thermal systems using the results obtained from the long-term optimal operation for the same system. In this study the system losses and the time delay of water flow may be taken into account.

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Appendix I

Cubic Splines

A spline approximation is a piecewise polynomial approximation. This means that a function f(x) is defined on an interval $a \leq x \leq b$, and we want to approximate f(x) on that interval by a function g(x) which is obtained as follows: We partition $a \leq x \leq b$, that is, we subdivide it into subintervals with common end points (nodes)

$$a = x_0 < x_1 < x_2 < \ldots < x_k = b,$$

and we require that g(x) in these subintervals is given by cubic polynomial, one polynomial per subinterval, such that at those endpoints g(x) is differentiable. Hence, instead of approximating f(x) by a single polynomial on the entire interval $a \leq x \leq b$, we now approximate f(x) by k polynomials. The cubic spline g(x) on $a \leq x \leq b$ is a continuous function which has continuous first and second derivatives evrywhere in that interval. Also, we have

$$g(x_0) = f(x_0),\ g(x_1) = f(x_1), \ldots, g(x_k) = f(x_k).$$

Hence, by using the cubic spline we guarantee that the interpolation of x(j) is exactly f(j) and the interpolation of $x(j+\delta)$ will be a suitable value of f on the cubic curve between the points j and j+1. This suits almost all the field measured tables we fit except two tables for Plants number 5/78 and 77/1018. Inthese two tables the field measured data given indicates a sharp change in tailwater variation against the total release. Hence, for these two curves, a cubic spline has been used in the first part which indicates a continuous first and second derivative and for the other part a linear curve fitting is used Fig. 12.